Appendix D

Alignment and Superelevation

A. Horizontal alignment

The operational characteristics of a roadway are directly affected by its alignment. The alignment, in turn, affects vehicle operating speeds, sight distances, and highway capacity. The horizontal alignment is influenced by many factors including:

- Terrain
- Functional classification
- Design speed
- Traffic volume
- Right-of-way availability
- Environmental concerns
- Anticipated level of service

The horizontal alignment must provide a safe, functional roadway facility that provides adequate sight distances within economical constraints. The alignment must adhere to specific design criteria such as minimum radii, superelevation rates, and sight distance. These criteria will maximize the overall safety of the facility and enhance the aesthetic appearance of the highway.

Construction of roadways along new alignments is relatively rare. Typically, roadways are reconstructed along existing alignments with horizontal and/or vertical changes to meet current design standards. The horizontal alignment of a roadway is defined in terms of straight-line tangents and horizontal curves. The curves allow for a smooth transition between the tangent sections. Circular curves and spirals are two types of horizontal curves utilized to meet the various design criteria.

1. Circular Curves

The most common type of curve used in a horizontal alignment is a simple circular curve. A circular curve is an arc with a single constant radius connecting two tangents. A compound curve is composed of two or more adjoining circular arcs of different radii. The centers of the arcs of the compound curves are located on the same side of the alignment. The combination of a short length of tangent between two circular curves is referred to as a broken-back curve. A reverse curve consists of two adjoining circular arcs with the arc centers located on opposite sides of the alignment. Compound and reverse curves are generally used only in specific design situations such as mountainous terrain.

Figure D-1 illustrates four examples of circular curves. The tangents intersect one another at the point of intersection (PI). The point at which the alignment changes from a tangent to circular section is the point of curvature (PC). The point at which the alignment changes from a circular to a tangent section is the point of tangency (PT). The point at which two adjoining circular curves turning in the same direction meet is the point of compound curvature (PCC). The point at which two adjoining circular curves turning in opposite directions meet is the point of reverse curvature (PRC).

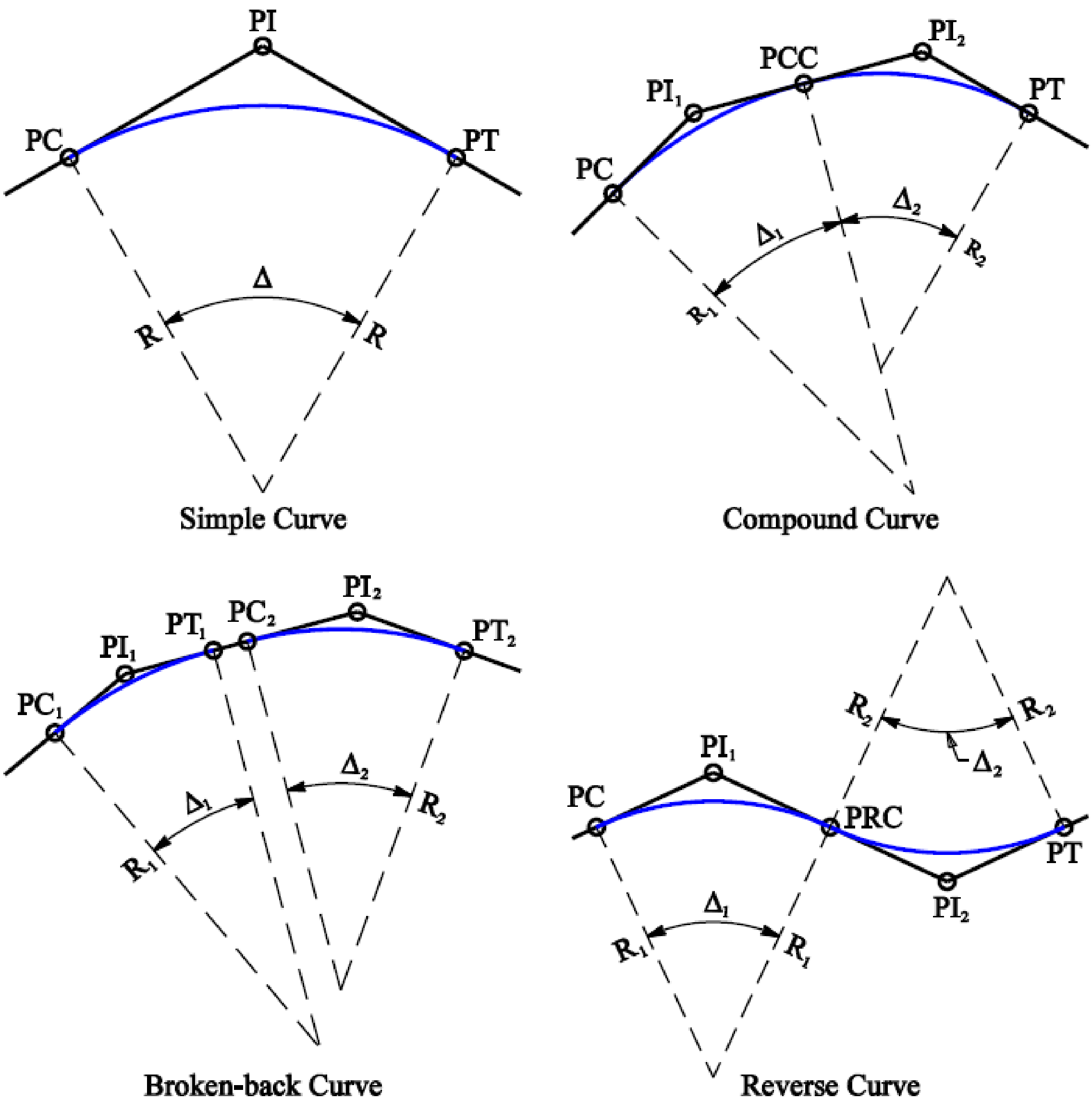

Figure D-1. Circular curves.

Figure D-2 is an illustration of the standard components of a single circular curve connecting a back and forward tangent. The distance from the PC to the PI is defined by the tangent distance (T). The length of the circular curve (L) is dependent on the central angle (Δ) and the radius (R) of the curve. Since the curve is symmetrical about the PI, the distance from the PI to the PT is also defined by the tangent distance (T). A line connecting the PC and PT is the long chord (LC). The external distance (E) is the distance from the PI to the midpoint of the curve. The middle ordinate (M) is the distance from the midpoint of the curve to the midpoint of the long chord.

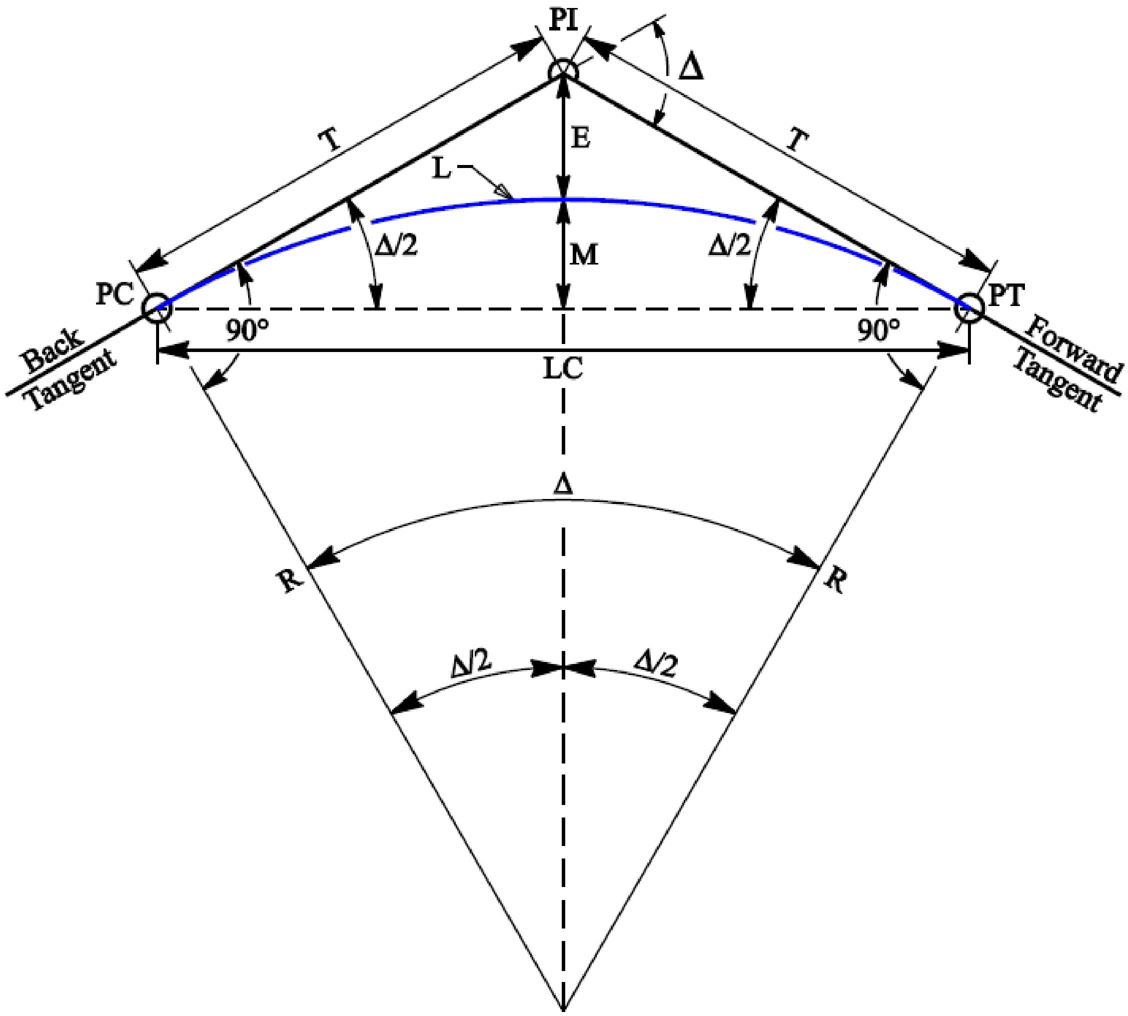

Figure D-2. Circular curve components.

Using the arc definition for a circular curve, the degree of curvature is the central angle (D) subtended by a 100 ft arc. A circle has an internal angle of 360° and a circumference of 2πR. Refer to Figure D-3 for an illustration of the degree of curvature within a circle. The relationship between the central angle and the radius for a given circular curve is:

$$\frac{D}{360°} = \frac{100\,ft}{2\pi R}; \quad D = \frac{(100\,ft)(360°)}{2\pi R} = \frac{5729.58\,ft}{R}$$

$$\frac{\Delta}{360°} = \frac{L}{2\pi R}; \quad \Delta = \frac{L * 360°}{2\pi R} = \frac{L * 180°}{\pi R}; \quad L = \frac{\Delta\pi R}{180°}$$

$$\frac{D}{\Delta} = \frac{100}{L}; \quad L = \frac{100\Delta}{D}$$

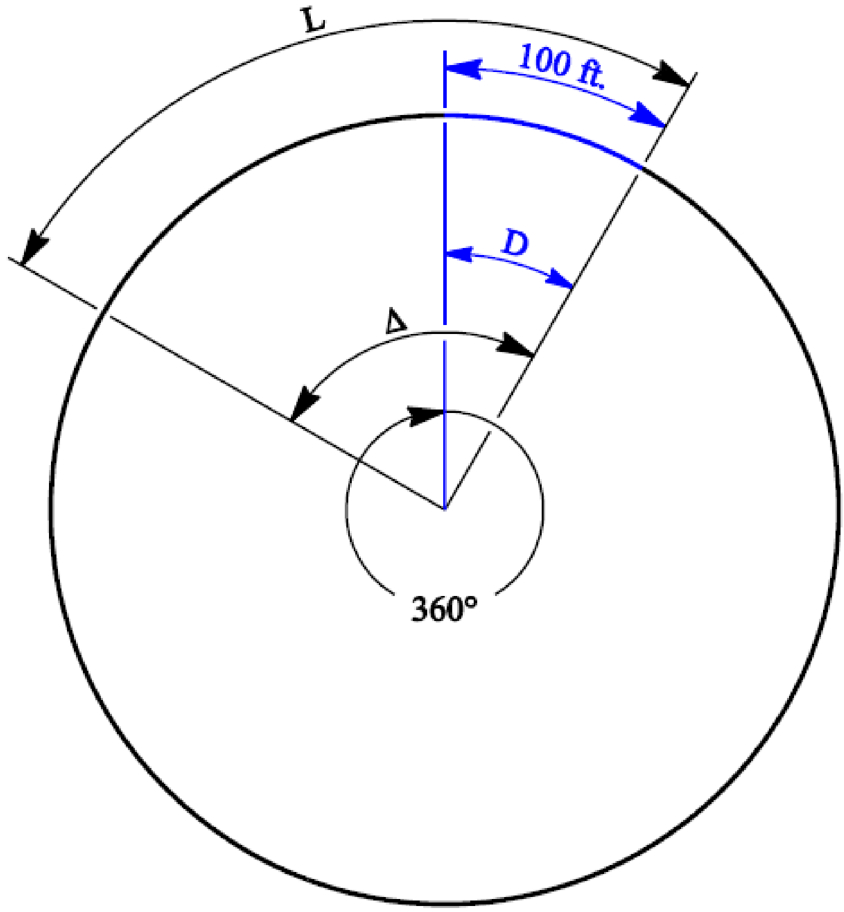

Figure D-3. Degree of curvature.

a. General Circular Curve Formulas

$$T = R \tan\frac{\Delta}{2}$$

$$L = \frac{100\Delta R}{5729.58} = \frac{100\Delta}{D}$$

$$LC = 2R \sin\frac{\Delta}{2}$$

$$E = \frac{R}{\cos\frac{\Delta}{2}} - R$$

$$M = E\cos\frac{\Delta}{2}$$

Stationing:

$$Sta.PC = Sta.PI - T$$

$$Sta.PT = Sta.PC + L$$

(1). Example Problem

Given: Sta. PI = 100+00
Radius = 4200 ft
Δ = 27°

Find: Sta. PT

Solution:

$$T = R\tan\frac{\Delta}{2} = 4200\tan\frac{27}{2} = 1008.33\ ft$$

$$Sta.PC = Sta.PI - T = 10000.00 - 1008.33 = 8991.67\ or\ 89 + 91.67$$

$$L = \frac{100\Delta R}{5729.58} = \frac{100 * 27 * 4200}{5729.58} = 1979.20\ ft$$

$$Sta.PT = Sta.PC + L = 8991.67 + 1979.20 = 10970.87\ or\ \mathbf{109 + 70.87}$$

b. Locating a point on a circular curve

The position of any point located at a distance l from the PC along a curve can be determined by utilizing the circular curve formulas.

$$t = R\tan\frac{\delta}{2}$$

$$D = \frac{5729.58}{R}$$

$$l = \frac{100\delta R}{5729.58} = \frac{100\delta}{D}$$

$$lc = 2R\sin\frac{\delta}{2}$$

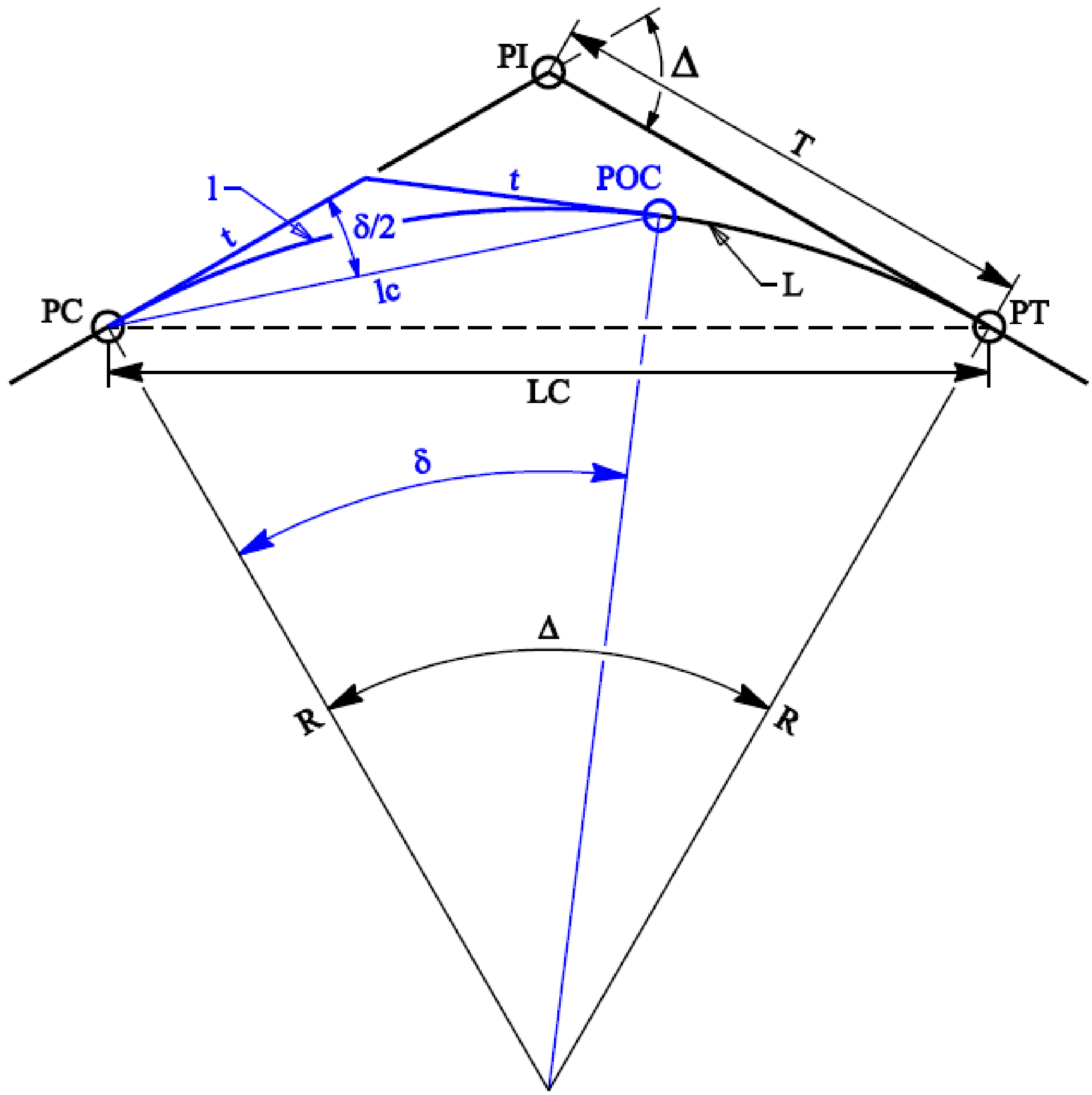

Figure D-4. Point on a circular curve.

2. Spiral Curves

Spiral curves are used in horizontal alignments to provide a gradual transition between tangent sections and circular curves. While a circular curve has a radius that is constant, a spiral curve has a radius that varies along its length. The radius decreases from infinity at the tangent to the radius of the circular curve it is intended to meet.

A vehicle entering a curve must transition from a straight line to a fixed radius. To accomplish this, the vehicle travels along a path with a continually changing radius. Consequently, a spiral will more closely duplicate the natural path of the turning vehicle. If the curvature of the alignment is not excessively sharp, the vehicle can usually traverse this spiral within the width of the travel lane. When the curvature is relatively sharp for a given design speed, it may become necessary to place a spiral transition at the beginning and end of the circular curve. The spirals allow the vehicle to more easily transition into and out of a curve while staying within the travel lane.

Figure D-5 illustrates the standard components of a spiral curve connecting tangents with a central circular curve. The back and forward tangent sections intersect one another at the PI. The alignment changes from the back tangent to the entrance spiral at the TS point. The entrance spiral meets the circular curve at the SC point. The circular curve meets the exit spiral at the CS point. The alignment changes from the exit spiral to the forward tangent at the ST point. The entrance and exit spiral at each end of the circular curve are geometrically identical.

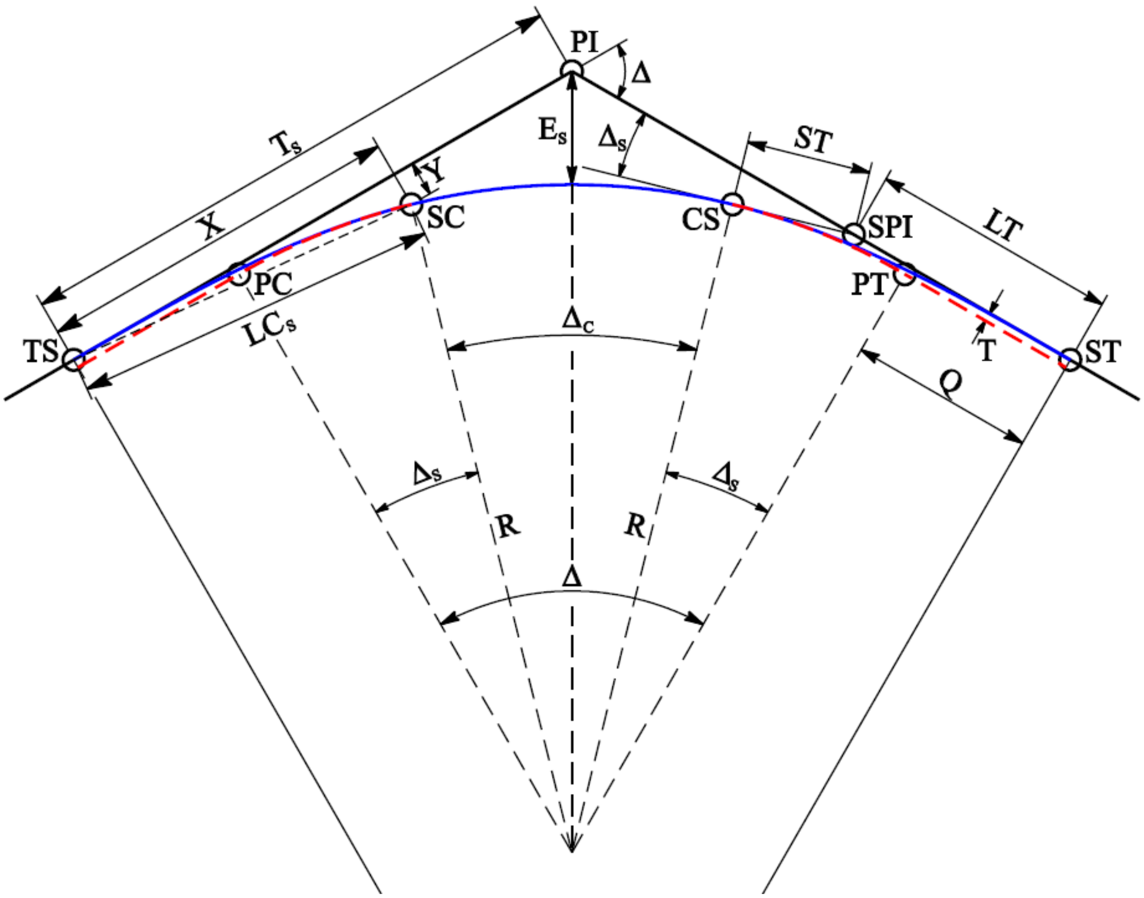

Figure D-5. Spiral curve components.

The length of the circular curve (L_C) is dependent on its central angle (Δ_C) and radius (R). The central angle (Δ) of the spiral-curve-spiral combination represents the deflection angle between the tangent sections. When spirals are placed at either end of the circular curve, the length of the curve is shortened. Instead of extending from the PC to the PT, the curve now extends from the SC to the CS. The offset distance or throw distance (T) represents the perpendicular distance from the back (or forward) tangent section to a tangent line extending from the PC (or PT) points. The length of the spiral (L_S) is typically determined by design speed and superelevation rates. The total length (L) of the spiral-curve-spiral combination is the sum of the length of curve (L_C) and the length of both spirals (L_S).

The distance from the TS to the PI is defined by the tangent distance (T_S). The external distance (E_S) is the distance from the PI to the midpoint of the circular curve. A line connecting the TS and SC (or the CS to the ST) is the long chord (LC_S) of the spiral. The Q dimension is the perpendicular distance from the TS to the PC (and the PT to the ST). The X dimension represents the distance along the tangent from the TS to the SC (and the CS to the ST). The Y dimension represents the tangent offset at the SC (and the CS). The LT and ST dimensions represent the long tangent and the short tangent of the spiral. The spiral tangents intersect at the spiral point of intersection (SPI).

a. General Spiral Equations

The central angle of a spiral (Δs) is a function of the average degree of curvature of the spiral. In other words, Δs of a spiral is one half of the central angle (Δ_C) for a circular curve of the same length and degree of curvature.

Since $\Delta_C = DL_C/100$ then $\Delta_S = DL_S/200$

$$\Delta = \Delta_C + 2\Delta_S$$

$$L = L_C + 2L_S$$

Spiral components such as X, Y, T, Q, ST, and LT are routinely found in spiral curve tables. Refer to Table D-1 for these values. These measurements are dependent on the spiral length (L_S) and central angle (Δ_S).

$$T_S = (R + T)\tan\frac{\Delta}{2} + Q$$

$$E_S = \frac{(R + T)}{\cos\frac{\Delta}{2}} - R$$

Stationing:

$$Sta.TS = Sta.PI - T_S$$

$$Sta.SC = Sta.TS + L_S$$

$$Sta.CS = Sta.SC + L_C$$

$$Sta.ST = Sta.CS + L_S$$

(1). Example Problem

Given: Sta. PI = 100+00
L_S = 150 ft
Δ = 35°
D = 10°

Find: Sta. TS, Sta. SC, Sta. CS, and Sta. ST

Solution:

$$R = \frac{5729.58}{D} = \frac{5729.58}{10} = 572.96\ ft$$

$$\Delta_S = \frac{DL_S}{200} = \frac{10 * 150}{200} = 7.5°$$

$$\Delta = \Delta_C + 2\Delta_S; \quad \Delta_C = \Delta - 2\Delta_S = 35° - 2 * 7.5° = 20°$$

$$L_C = \frac{100\Delta c}{D} = \frac{100 * 20}{10} = \frac{2000}{10} = 200.00\ ft$$

The following values were determined from Table D-1 for a spiral central angle (Δ_S) of 10° 00' 00":

X = 149.74', Y = 6.54', T = 1.64', Q = 74.96', ST = 50.08', and LT = 100.09'.

$$T_S = (R + T)\tan\frac{\Delta}{2} + Q = (572.96 + 1.64)\tan\frac{35}{2} + 74.96 = 256.13\ ft$$

$$Sta.TS = Sta.PI - T_S = 10000.00 - 256.13 = 9743.87\ or\ \mathbf{97 + 43.87}$$

$$Sta.SC = Sta.TS + L_S = 9843.87 + 150.00 = 9893.87\ or\ \mathbf{98 + 93.87}$$

$$Sta.CS = Sta.SC + L_C = 9893.87 + 200.00 = 10093.87\ or\ \mathbf{100 + 93.87}$$

$$Sta.ST = Sta.CS + L_S = 10093.87 + 150.0 = 10243.87\ or\ \mathbf{102 + 43.87}$$

D	Δ_S	R	X	Y	T	Q	LC_S	ST	LT
7° 30' 00"	5° 37' 30"	763.94	149.86	4.91	1.23	74.98	149.94	50.05	100.05
8° 00' 00"	6° 00' 00"	716.20	149.84	5.23	1.31	74.97	149.93	50.05	100.06
8° 30' 00"	6° 22' 30"	674.07	149.81	5.56	1.39	74.97	149.92	50.06	100.06
9° 00' 00"	6° 45' 00"	636.62	149.79	5.88	1.47	74.97	149.91	50.07	100.07
9° 30' 00"	7° 07' 30"	603.11	149.77	6.21	1.55	74.96	149.90	50.07	100.08
10° 00' 00"	7° 30' 00"	572.96	149.74	6.54	1.64	74.96	149.89	50.08	100.09
10° 30' 00"	7° 52' 30"	545.67	149.72	6.86	1.72	74.95	149.87	50.09	100.10
11° 00' 00"	8° 15' 00"	520.87	149.69	7.19	1.80	74.95	149.86	50.10	100.11
11° 30' 00"	8° 37' 30"	498.22	149.66	7.51	1.88	74.94	149.85	50.11	100.12
12° 00' 00"	9° 00' 00"	477.46	149.63	7.84	1.96	74.94	149.84	50.12	100.13
13° 00' 00"	9° 45' 00"	440.74	149.57	8.49	2.12	74.93	149.81	50.14	100.15
14° 00' 00"	10° 30' 00"	409.26	149.50	9.14	2.29	74.92	149.78	50.16	100.18
15° 00' 00"	11° 15' 00"	381.97	149.42	9.79	2.45	74.90	149.74	50.18	100.20
16° 00' 00"	12° 00' 00"	358.10	149.34	10.44	2.61	74.89	149.71	50.21	100.23
17° 00' 00"	12° 45' 00"	337.03	149.26	11.09	2.78	74.88	149.67	50.24	100.26
18° 00' 00"	13° 30' 00"	318.31	149.17	11.73	2.94	74.86	149.63	50.27	100.29

Table D-1. Spiral table for L_S = 150 ft.

b. Locating a point on a spiral curve

The position of any point located at a distance l from the TS along a spiral can be determined by modifying the spiral curve formulas.

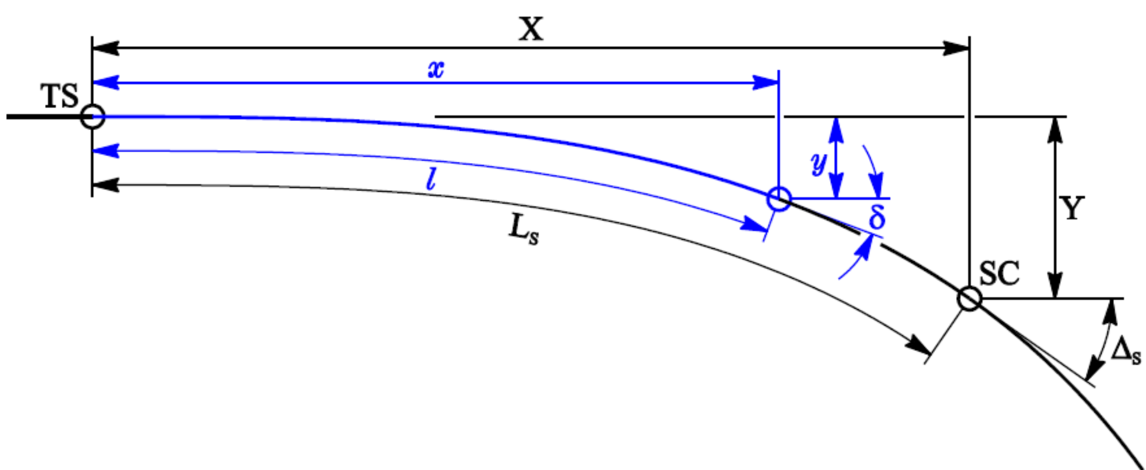

Figure D-6. Point on a spiral curve.

The deflection angle (δ) at the intermediate point can be determined by the equation:

$$\delta = \Delta_S \frac{l^2}{{L_S}^2}$$

Using the equation for determining the spiral central angle equation, δ can also be solved by:

$$\Delta_S = \frac{DL_S}{200}; \quad \delta = \frac{Dl^2}{200L_S}$$

By using differential geometry and an infinite series for the sine and cosine functions, the distance along the tangent (x) and the tangent offset (y) can be determined. In the following equations, δ must be converted to radians by multiplying the angle in degrees by π/180.

$$x = l(1 - \frac{\delta^2}{(5)(2!)} + \frac{\delta^4}{(9)(4!)} - \frac{\delta^6}{(13)(6!)} + \cdots)$$

$$y = l(\frac{\delta}{3} - \frac{\delta^3}{(7)(3!)} + \frac{\delta^5}{(11)(5!)} - \frac{\delta^7}{(15)(7!)} + \cdots)$$

The X and Y values can be calculated by substituting L_S for l, and Δ_S for δ. After X and Y have been determined, the following values can be calculated using these equations:

$$Q = X - R\sin\Delta_S; \quad T = Y - R(1 - \cos\Delta_S)$$

$$ST = \frac{Y}{\sin\Delta_S}; \quad LT = X - ST\cos\Delta_S$$

$$LC_S = \sqrt{(X\cos\Delta_S + Y\sin\Delta_S)^2 + (X\sin\Delta_S - Y\cos\Delta_S)^2}$$

The spiral deflection between the tangent section and the spiral long chord is approximately ⅓ of the spiral deflection angle (Δ_S). By using these substitutions, the calculations for the spiral components may be greatly simplified.

$$Y = L_S * \sin\frac{\Delta_S}{3}; \quad X^2 + Y^2 = L_S{}^2 \; or \; X = \sqrt{L_S{}^2 - Y^2}$$

$$Q = \frac{X}{2}; \quad T = \frac{Y}{4}$$

$$ST = \sin\frac{\Delta_S}{3} * \frac{L_S}{\sin\Delta_S}; \quad LT = \sin\frac{2\Delta_S}{3} * \frac{L_S}{\sin\Delta_S}$$

B. Superelevation

Centrifugal force is the outward pull on a vehicle traversing a horizontal curve. When traveling at low speeds or on curves with large radii, the effects of centrifugal force are minor. However, when travelling at higher speeds or around curves with smaller radii, the effects of centrifugal force increase. Excessive centrifugal force may cause considerable lateral movement of the turning vehicle and it may become impossible to stay inside the driving lane.

Superelevation and side friction are the two factors that help stabilize a turning vehicle. Superelevation is the banking of the roadway such that the outside edge of pavement is higher than the inside edge. The use of superelevation allows a vehicle to travel through a curve more safely and at a higher speed than would otherwise be possible. Side friction developed between the tires and the road surface also acts to counterbalance the outward pull on the vehicle. Side friction is reduced when water, ice, or snow is present or when tires become excessively worn.

The transitional rate of applying superelevation into and out of curves is influenced by several factors. These factors include design speed, curve radius, and number of travel lanes. Minimum curve radii for a horizontal alignment are determined by the design speed and superelevation rate. Higher design speeds require more superelevation than lower design speeds for a given radius. Additionally, sharper curves require more superelevation than flatter curves for a given design speed.

The maximum superelevation for a section of roadway is dependent on climatic conditions, type of terrain, and type of development. Roadways in rural areas are typically designed with a maximum superelevation rate of 8 percent. In mountainous areas, a maximum superelevation rate of 6 percent is used due to the increased likelihood of ice and snow. Urban roadways are normally designed with a maximum superelevation rate of 4 percent. Superelevation is of limited use in urban areas because of the lower operating speeds. In many cases, superelevation in urban areas may be completely eliminated. The superelevation of the roadway may interfere with drainage systems, utilities, and pavement tie-ins at intersecting streets and driveways.

Superelevation is gradually introduced by rotating the pavement cross-section about a point of rotation. For undivided highways, the point of rotation is located at the centerline. For divided highways, the point of rotation is typically located at the inside edge of traveled way. The location of the point of rotation is generally indicated on the roadway typical sections. Superelevation is applied by first rotating the lane(s) on the outside of the curve. The inside lane(s) do not rotate until the outside lane(s) achieve a reverse crown. At this point, all lanes rotate simultaneously until full superelevation is reached.

The length of crown runoff (C) is the distance required for the outside lane(s) to transition from a normal crown to a flat crown. The length of crown runoff is also the distance for the outside lane(s) to transition from a flat crown to a reverse crown. The length of the superelevation runoff (S) is the distance required for the transition from a flat crown to the full superelevation rate (e). The values of C and S are determined from superelevation tables for various combinations of design speed and degree of curvature. Refer to Table D-2 to view a portion of the WYDOT superelevation tables located in the Roadway Design Manual.

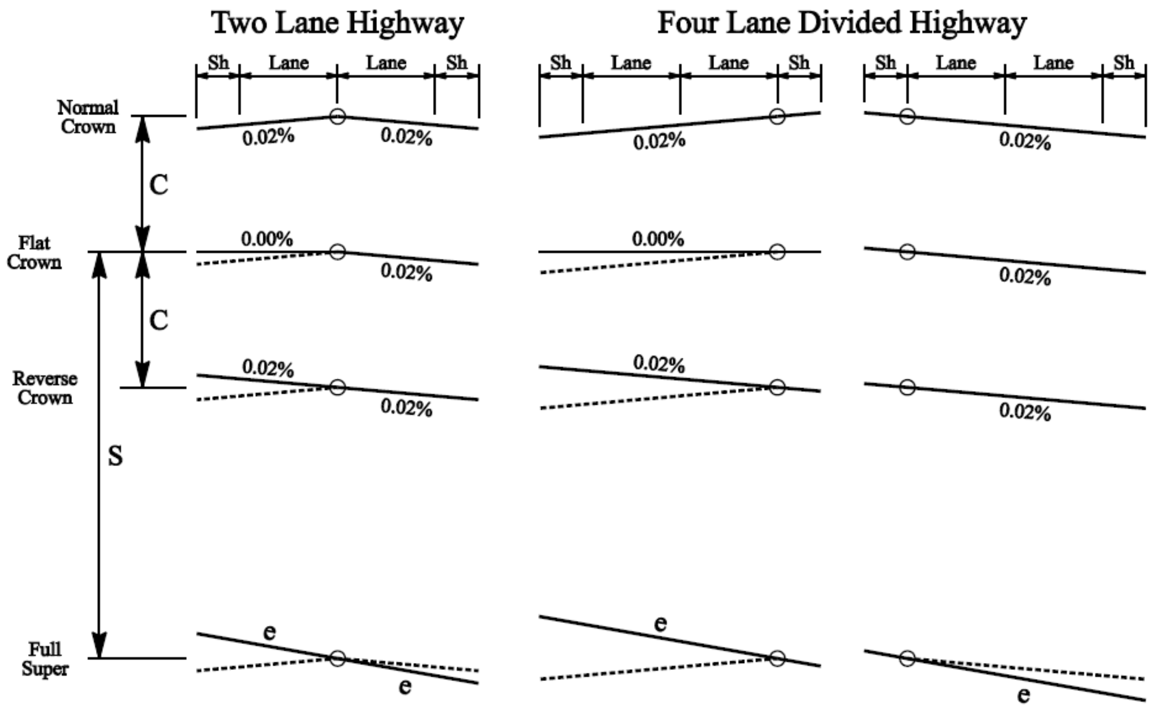

Figure D-7. Superelevation rotation.

1. Circular Curves

Superelevation is uniformly applied to provide a smooth transition from a normal crown section to a full superelevation section. Two-thirds of superelevation runoff occurs prior to the PC and then again after the PT. One-third of the superelevation runoff occurs on the curve between the PC and the PT at each end of the curve. The rest of the curve is in a full superelevation section. The crown runoff that transitions from a normal crown to a flat crown (and vice versa) is placed outside each superelevation runoff section. The crown runoff that transitions from a flat crown to a reverse crown (and vice versa) is placed just inside each superelevation runoff section. See Figure D-8 for an illustration of the crown and superelevation runoff distances as they are applied to circular curves.

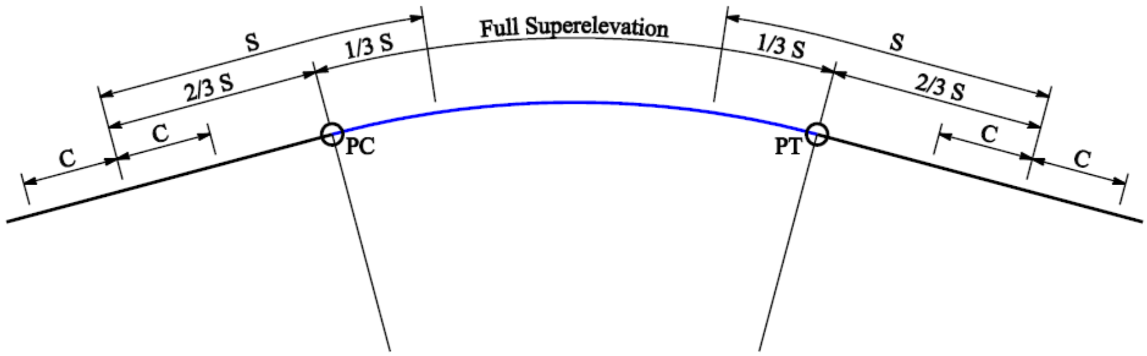

Figure D-8. Superelevation on a circular curve.

2. Spiral Curves

Where spiral transition curves are used, the full length of the spiral is equal to the superelevation runoff. The full superelevation is reached at the SC point and the entire circular curve is in a full superelevation section. The crown runoffs that transition from a normal crown to a flat crown (and vice versa) occurs prior to the TS point and after the ST point. The crown runoff that transitions from a flat crown to a reverse crown is placed just after the TS point and before the ST point. See Figure D-9 for an illustration of the crown and superelevation runoff distances as they are applied to spirals curves.

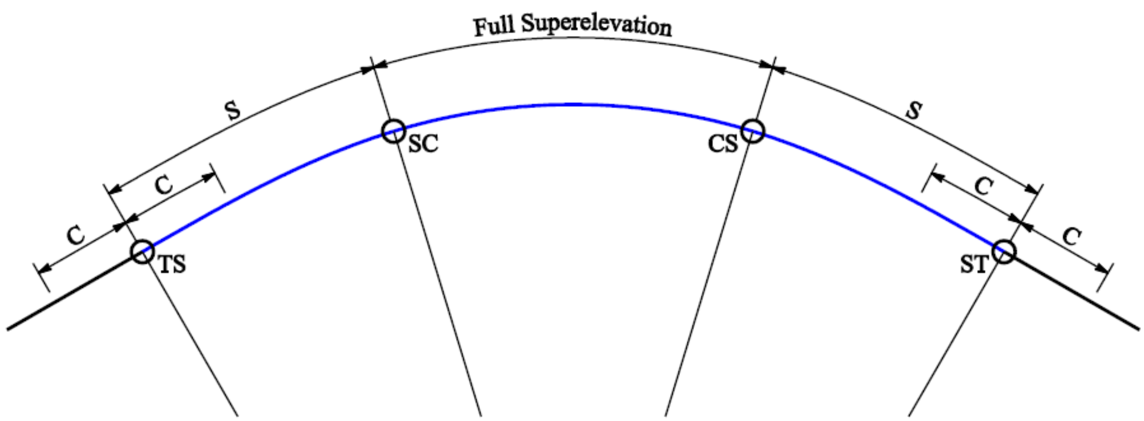

Figure D-9. Superelevation on a spiral/circular curve combination.

3. Compound and Reverse Curves

Because of their unique configuration, compound and reverse curves demand careful consideration when applying superelevation. These situations are outside the scope of this manual and the WYDOT Road Design Manual should be consulted.

DESIGN - HORIZONTAL ALIGNMENT AND SUPERELEVATION

WYDOT Superelevation Tables

RURAL DESIGN e(max) = 0.08 ft/ft

Lane Adjustment Factor L(adj) = 1.5

(For other lane adjustment factors, divide S and C by 1.5 and multiply by desired lane adjustment factor.)

Rev. 8-9-00

e(max)= 0.08 L(adj) = 1.5		e(max) 0.08, f(max) 0.18, V(R) 15, V(D) 15				e(max) 0.08, f(max) 0.17, V(R) 20, V(D) 20			
CURVE		**15 mph** (24km/h)				**20 mph** (32km/h)			
R (ft)	D (deg)	e (ft/ft)	S (ft)	C (ft)	T (ft)	e (ft/ft)	S (ft)	C (ft)	T (ft)
23000	0.25	NC	0	0	0.0	NC	0	0	0.0
20000	0.29	NC	0	0	0.0	NC	0	0	0.0
17000	0.34	NC	0	0	0.0	NC	0	0	0.0
14000	0.41	NC	0	0	0.0	NC	0	0	0.0
12000	0.48	NC	0	0	0.0	NC	0	0	0.0
10000	0.57	NC	0	0	0.0	NC	0	0	0.0
8000	0.72	NC	0	0	0.0	NC	0	0	0.0
6000	0.95	NC	0	0	0.0	NC	0	0	0.0
5000	1.15	NC	0	0	0.0	NC	0	0	0.0
4000	1.43	NC	0	0	0.0	NC	0	0	0.0
3500	1.64	NC	0	0	0.0	NC	0	0	0.0
3000	1.91	NC	0	0	0.0	NC	0	0	0.0
2500	2.29	NC	0	0	0.0	NC	0	0	0.0
2000	2.86	NC	0	0	0.0	NC	0	0	0.0
1800	3.18	NC	0	0	0.0	NC	0	0	0.0
1600	3.58	NC	0	0	0.0	RC	45	45	0.1
1400	4.09	NC	0	0	0.0	RC	45	45	0.1
1200	4.77	NC	0	0	0.0	RC	45	45	0.1
1000	5.73	NC	0	0	0.0	0.024	55	46	0.1
900	6.37	RC	40	40	0.1	0.026	60	46	0.2
800	7.16	RC	40	40	0.1	0.029	65	45	0.2
700	8.19	RC	40	40	0.1	0.032	75	47	0.3
600	9.55	0.022	45	41	0.1	0.036	85	47	0.5
500	11.46	0.026	55	42	0.3	0.041	95	46	0.8
450	12.73	0.029	60	41	0.3	0.044	100	45	0.9
400	14.32	0.032	65	41	0.4	0.048	110	46	1.3
350	16.37	0.035	75	43	0.7	0.052	120	46	1.7
300	19.10	0.039	80	41	0.9	0.056	130	46	2.3
250	22.92	0.045	95	42	1.5	0.061	140	46	3.3
200	28.65	0.051	105	41	2.3	0.067	155	46	5.0
150	38.20	0.059	125	42	4.3	0.075	175	47	8.4
100	57.30	0.070	145	41	8.6				
75	76.39	0.077	160	42	13.6				
50	114.59								
		R(min) = 59 ft				R(min) = 107 ft			

Definitions for Superelevation Tables:

R = Radius of Curve
D = Degree of Curve (100 ft arc length definition)
e = Superelevation Rate
S = Length of Superelevation Runoff & Spiral Length
Flat Crown to full superelevation length.
WYDOT uses 2/3 S off curve, 1/3 S on curve.
C = Length of Crown Runoff
NC (normal crown) to flat crown length.
T = Spiral Throw Distance
Spirals are recommended when T=>2 ft.
NC = Normal Crown (typically 0.02 ft/ft)
RC = Reverse Crown (typically 0.02 ft/ft)
L(adj) = Length adjustment Factor
Used to adjust the length of S and C for multilane highways.
L(adj) = 1.0 AASHTO & WYDOT Min. for two lane roadways rotated about centerline.
L(adj) = 1.5 WYDOT "desirable" for two lane roadways rotated about centerline.
L(adj) = 1.5 AASHTO & WYDOT Min. and "desirable" for four lane divided highways rotated about the inside edge of traveled way.
See Road Design Manual for other situations and values of L(adj).

Superelevation Values Based on AASHTO 2000 Greenbook.

S & C values based on WYDOT Edgeline Gradient formula: Grad = V(D) * 2.4 +80
Where V(D) = Design Speed in mph
and Grad is the ratio of 1 : Grad
S & C values may vary slightly from AASHTO values. Also 2-sec. min. is no longer required by AASHTO and has been eliminated from the tables.

1. Spiral Transitions are strongly recommended when T is equal to or greater than 2 ft. Table values for e, S, C and T are shaded when spirals should be used.
2. When necessary, L(adj) may be reduced to 1.0 for two lane roadways rotated about centerline.
3. Consideration of reverse crown should be given when the tables call for NC (normal crown).

Table D-2. WYDOT superelevation table.

C. Vertical Alignment

The vertical alignment of a roadway is controlled by design speed, topography, traffic volumes, highway functional classification, sight distance, horizontal alignment, vertical clearances, drainage, economics, and aesthetics. The vertical alignment (or profile) of the roadway is typically established at the point of rotation. This point is used because it is not affected by superelevation.

Vertical curves are used to provide a smooth transition between roadway grades. A vertical curve is composed of a parabolic curve that provides a constant rate of change of grade. A parabolic curve "flattens" the curve at the top or bottom of a hill to maximize the driver's sight distance.

There are several factors that will influence the vertical alignment of a roadway. Matching the proposed grade with the existing terrain will reduce the depths of cuts and fills. Balancing the volume of cut material with fill material minimizes the need for borrow sources or waste areas. The vertical alignment should not exceed maximum grades, nor should it interfere with existing drainage structures. The vertical alignment will have to meet fixed elevations to accommodate passing over, passing under, or intersecting with other roadways or railroads. Lastly, the vertical curve should be of sufficient length to provide sufficient sight distance.

There are two basic types of vertical curves, crest and sag. As the name implies, a crest vertical curve occurs at the top of the slope and a sag vertical curve occurs at the bottom of a slope. The grade lines intersect one another at the vertical point of intersection (VPI). The slope of the first grade line is labeled G_1 and the slope of the second grade line is labeled G_2. When the grade rises along the alignment, G is positive, when the grade falls or slopes downward, G is negative. The grade line G is always expressed in terms of a percentage. The algebraic difference of grades is expressed as A.

The vertical curve begins at the (BVC) point and ends at the (EVC) point. The length (L) of the vertical curve is the projection of the curve onto a horizontal surface. In most cases, the vertical curve will have equal-tangent lengths (i.e. the VPI is located midway between the BVC and the EVC). The high point in a crest curve and the low point in a sag curve are defined by a horizontal distance (x) from the BVC point. The elevation at the high or low point (y) is the vertical distance above a datum. The high and low points are important locations on a vertical curve. The high point on a crest curve must be identified to allow for minimum overhead clearances to an overpass or utility line. The low point on a sag curve must be identified to allow for minimum amount of cover over drainage structures. Refer to Figure D-10 for an illustration of the components of a crest and sag vertical curve.

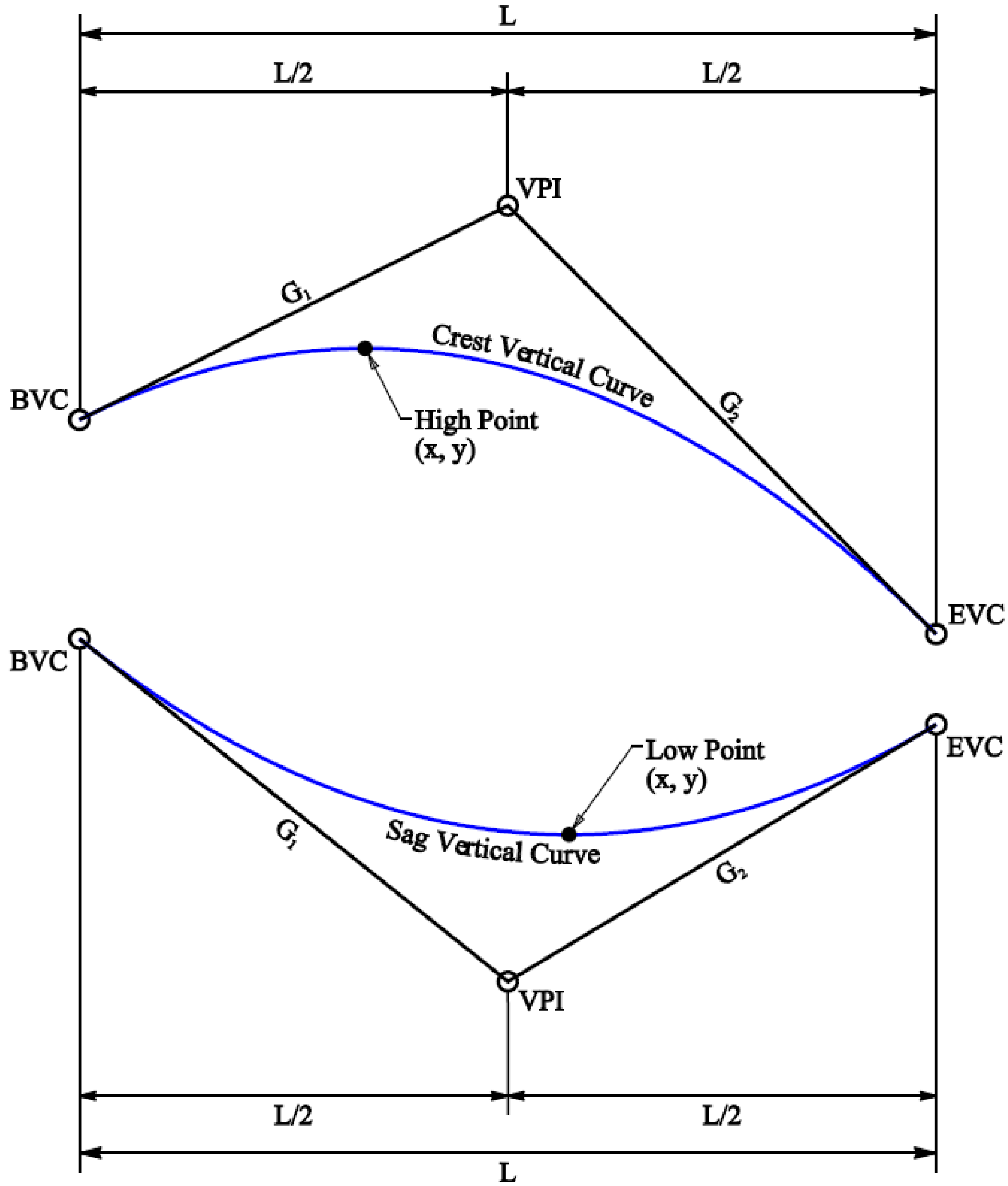

Figure D-10. Crest and sag vertical curves.

1. General Vertical Curve Equations

The general equation for a parabola is:

$$y = ax^2 + bx + c$$

Where y is the elevation at any point along the parabola located a distance x from the BVC point. The values for a, b, and c are constants.

The slope of the curve at any point is determined by the first derivative of the parabolic equation:

$$\frac{dy}{dx} = 2ax + b$$

The rate of change of grade is determined by the second derivative of the parabolic equation:

$$\frac{d^2x}{d^2y} = 2a$$

The rate of change of grade (2a) can also be written as A/L.

$$2a = \frac{A}{L}; \quad or\ a = \frac{A}{2L}$$

The slope of a tangent at the beginning of the curve (b) is equal to G_1. The distance of the BVC above or below a vertical datum is expressed as c. Since A is the algebraic difference of grade:

$$A = G_2 - G_1$$

The parabolic equation can be rewritten as:

$$y = \frac{A}{2L} * x^2 + G_1 * x + BVC_{ELEV} = \frac{(G_2 - G_1)}{2L} * x^2 + G_1 * x + BVC_{ELEV}$$

At the high or low point of a parabola, the slope is equal to zero. To solve for the x and y value at the high or low point of a vertical curve, use the following set of equations:

$$Slope = 2ax + b; \quad or\ Slope = 2ax + G_1$$

Since the slope at the high or low point is equal to zero:

$$0 = 2ax + G_1$$

$$2ax = -G_1$$

$$x = -\frac{G_1}{2a}; \quad since\ a = \frac{A}{2L}; \quad x = -\frac{G_1}{2(A/2L)} = -\frac{G_1L}{A}$$

Stationing:

$$Sta.BVC = Sta.PVI - {}^{L}/_{2}$$

$$Sta.EVC = Sta.PVI + {}^{L}/_{2}$$

a. Example Problem

Given: Sta. BVC = 30+30
L = 300 ft
G_1 = -3.2%
G_2 = 1.8%
BVC Elev. = 4165.92 ft

Find: The elevations and stations at the low point, PVI, and EVC.

Solution:

$$A = G_2 - G_1 = 1.8 - (-3.2) = 5.0$$

$$Sta.PVI = Sta.BVC + L/2 = 3030.00 + 300/2 = 3180.00\ or\ \mathbf{31 + 80.00}$$

$$Sta.EVC = Sta.PVI + L/2 = 3180.00 + 300/2 = 3330.00\ or\ \mathbf{33 + 30.00}$$

$$PVI\ Elev. = BVC\ Elev. + L/2 * G_1 = 4165.92 + 300/2 * (-0.032) = \mathbf{4161.12\ ft}$$

$$EVC\ Elev. = PVI\ Elev. + L/2 * G_2 = 4161.12 + 300/2 * (0.018) = \mathbf{4163.82\ ft}$$

Low point location:

$$x = -\frac{G_1 L}{A} = -\frac{(-0.032) * 300}{0.05} = \mathbf{192.00\ ft}$$

$$Low\ pt.\ Sta. = Sta.BVC + x = 3030.00 + 192.00 = 3222.00\ or\ \mathbf{32 + 22.00}$$

$$y = \frac{(G_2 - G_1)}{2L} * x^2 + G_1 * x + BVC_{ELEV}$$

$$y = \frac{(0.018 - (-0.032))}{2 * 300} * (192)^2 - 0.032 * 192 + 4165.92 = \mathbf{4162.85\ ft}$$

2. Locating a point on a vertical curve

The position of any point located at a distance x from the BVC along the curve can be determined by utilizing the vertical curve formulas.

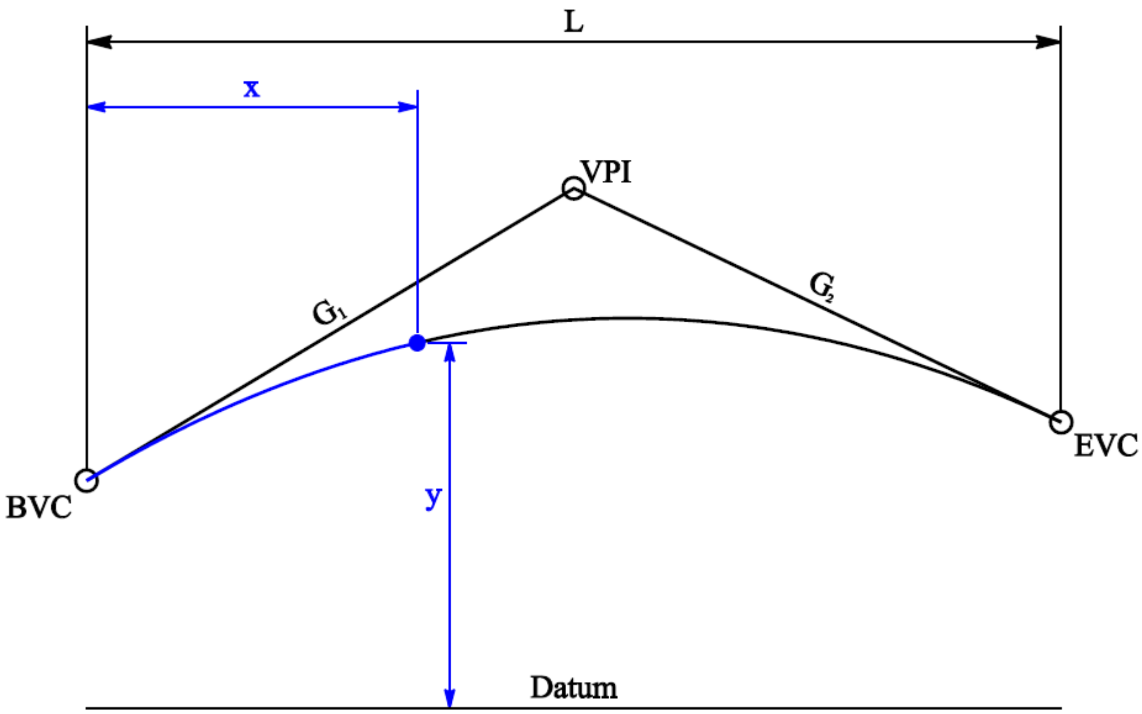

Figure D-11. Point on a vertical curve.

The horizontal distance (x) from the BVC is the first determination. Next, the curve elevation (y) is calculated:

$$y = \frac{A}{2L} * x^2 + G_1 * x + BVC_{ELEV}$$

An elevation along the tangent from the BVC to the PVI can be calculated by the following formula:

$$Tangent\ Elevation = G_1 * x + BVC_{ELEV}$$

Tangent elevations from the PVI to the EVC are determined by the following formula:

$$Tangent\ Elevation = G_2 * (x - L/2) + VPI_{ELEV}$$

Refer to Table D-3 for a spreadsheet using these equations to calculate the elevations along both tangents and the curve at specific intervals. The vertical curve used for the calculation in Table D-3 has a length of 600 ft, an initial grade of 3.00%, a second grade of -2.40%, and an elevation at the BVC of 5128 ft.

	Station	Distance from BVC (x)	Tangent Elevation	Curve Elevation (y)
BVC	23+85.00	0.00	5128.00	5128.00
	24+00.00	15.00	5128.45	5128.44
	24+50.00	65.00	5129.95	5129.76
	25+00.00	115.00	5131.45	5130.85
	25+50.00	165.00	5132.95	5131.72
	26+00.00	215.00	5134.45	5132.37
	26+50.00	265.00	5135.95	5132.79
PVI	26+85.00	300.00	5137.00	5132.95
	27+00.00	315.00	5136.64	5132.98
Low Point	27+18.33	333.33	5136.20	5133.00
	27+50.00	365.00	5135.44	5132.95
	28+00.00	415.00	5134.24	5132.70
	28+50.00	465.00	5133.04	5132.22
	29+00.00	515.00	5131.84	5131.51
	29+50.00	565.00	5130.64	5130.58
EVC	29+85.00	600.00	5129.80	5129.80

Table D-3. Vertical curve elevations.

3. Unsymmetrical Vertical Curves

This type of vertical curve is utilized when a standard symmetrical vertical curve cannot be made to fit the terrain. Essentially, the vertical curve consists of two unequal tangent lengths. In other words, the horizontal distance from the BVC to the VPI is not equal to the horizontal distance from the VPI to the EVC. The end of the first curve is also the beginning of the second curve. This point is referred to as the point of compound vertical curvature (CVC). Refer to Figure D-12 for an illustration of an unsymmetrical vertical curve.

An unsymmetrical vertical curve is comprised of two symmetrical curves. The calculations required to determine the curve and tangent elevations are the same as for a single symmetrical curve. However, an additional grade line needs to be determined. A line drawn tangent to the curve at the CVC point is equal to G_2 for the first vertical curve and G_1 for the second. This grade line is parallel to the slope of the grade from the BVC of the first curve and the EVC of the second. The slope of this grade line is determined by the following formula:

$$G = (EVC_{ELEV} - BVC_{ELEV})/L$$

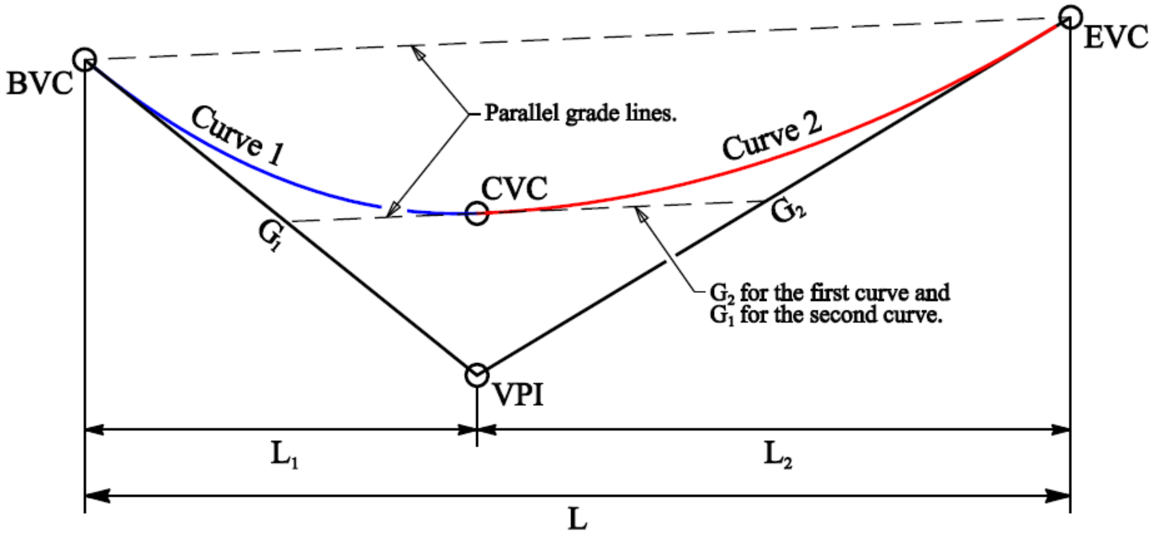

Figure D-12. Unsymmetrical vertical curve.