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Systematic back-calculation protocol and prediction of resilient modulus for MEPDG

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ABSTRACT

A research focusing on the characterisation of representative local material properties was conducted to facilitate the full implementation of the Mechanistic-Empirical Pavement Design Guide for roadway designs in Wyoming. As part of the test program, falling weight deflectometer deflection data were collected from 25 test sites in Wyoming for back-calculation of subgrade resilient modulus. Also, subgrade materials from these test sites were sampled for laboratory resilient modulus measurement in accordance with the AASHTO T 307. The back-calculation is a user-dependent procedure and produces a non-unique resilient modulus estimation. To alleviate this limitation, this paper focuses on the recent development of a systematic back-calculation protocol for subgrade resilient modulus using MODCOMP6 software. The protocol is intended for use on a flexible pavement with a crushed base. The proposed procedure discusses pre-analysis checks, seed modulus adjustment, pavement structure adjustment and program termination criteria. A correlation study was conducted to correct back-calculated resilient modulus to laboratory-equivalent values. The results conclude that a non-zero intercept linear regression model provides a better correlation than the widely used zero intercept linear regression model. Furthermore, better correlations are achieved when the back-calculated resilient modulus of a lower subgrade layer and resilient modulus measured at higher laboratory test sequences Nos. 11 to 15 are considered. The non-zero model based on M_r test sequence No. 14 and lower subgrade layer yields the best correlation. For the zero model, a C-factor of 0.645 based on M_r test sequence No. 15 and lower subgrade layer yields the best correlation.

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KEYWORDS

Pavement; MEPDG; back-calculation; resilient modulus; falling weight; deflectometer

1. Introduction

The continuous increases in traffic volumes and loadings, as well as the need to account for varying climatic and material effects, have led to the development of Mechanistic-Empirical Pavement Design Guide (MEPDG) under the National Cooperative Highway Research Program (NCHRP) 1-37A in 2004. The MEPDG intends to replace the American Association of State and Highway Transportation Officials (AASHTO) Guide for Design of Pavement Structures (AASHTO 1993). Pavement performance-based designs are accomplished from the MEPDG utilising mechanistic empirical (M-E) models to predict pavement distresses. In so doing, the MEPDG requires an in-depth analysis of over hundreds to thousands of inputs for both flexible and rigid pavements based on local traffic, climate and material conditions. However, the M-E models were initially developed based on nationwide long-term pavement performance (LTPP) data that do not necessarily represent a local condition. To facilitate higher level hierarchical design and to obtain better design accuracy, a local calibration of MEPDG was recommended by the AASHTO. Local calibration can be accomplished through either a comprehensive test program to collect project-level local data or the implementation of pavement performance data available from the pavement management system.

As the state of Wyoming, USA, makes the transition from the AASHTO Guide for Design of Pavement Structures (AASHTO 1993) to the MEPDG, particular interest has been focused on the characterisation of design inputs based on local traffic, climate and material conditions. Traffic inputs have been addressed by the Applied Research Associates (ARA) Inc., utilising weigh-in-motion sensors throughout Wyoming to collect traffic information. Climate data from the National Climatic Data Center and three weather stations were considered to provide local climatic input values (Dzotepe and Ksaibati 2010). To facilitate the full implementation of the MEPDG in the state of Wyoming, the current research project focuses on the quantification of local soil properties, especially the resilient modulus of unbound subgrade materials. Resilient modulus is one of the primary soil property inputs for a subgrade material to compute stresses, strains and deformations induced in a pavement structure by an applied traffic load. Correctly determining the subgrade resilient modulus can significantly affect the required thicknesses of the pavement layers and directly influence the cost. If inaccurate resilient modulus values are used, pavement structures can be overdesigned leading to increased costs, or under-designed leading to a premature failure. This paper particularly focuses on the back-calculation of resilient moduli (M_r) of unbound subgrade

materials based on a newly developed systematic protocol. The back-calculated M_R values are then corrected to match laboratory resilient modulus (M_r) values measured in accordance with a modified AASHTO T-307 test procedure proposed for the state of Wyoming (Henrichs 2015). The development of constitutive models to estimate the subgrade resilient modulus in terms of regression coefficients (k_1 , k_2 and k_3) was discussed by Henrichs (2015).

The conventional M_R back-calculation is an iterative process by which the moduli of pavement structure layers are simultaneously estimated using the corresponding deflection data measured by a falling weight deflectometer (FWD) (Alavi *et al.* 2008). Several back-calculation methods have been developed for the flexible pavement. Dong *et al.* (2001) used a 3D finite element method to determine the time-domain back-calculation of pavement properties. Goktepe *et al.* (2006) considered the static effects of the FWD deflection on back-calculation, while Seo *et al.* (2009) considered its dynamic effects. Gopalakrishnan and Papadopoulos (2011) applied a novel machine learning concept and Saltan *et al.* (2011) used a data mining method in the pavement back-calculation. Although the FWD approach is considered a preferred nondestructive testing method that offers many advantages over laboratory M_r testing, especially with its lower cost and higher testing efficiency, its key limitation lies in the back-calculation process. The back-calculation is a user-dependent procedure that requires adequate knowledge of the pavement structure and material properties. It produces a non-unique solution for each test because of the indeterminate nature of the analysis (i.e. number of unknown variables is larger than the number of available solving equations).

A study was completed by Dawson *et al.* (2009) for the Michigan Department of Transportation to compare back-calculated (M_R) and laboratory-measured (M_r) resilient modulus values of subgrade soils. They concluded that a relatively good agreement between M_R and M_r values was obtained when the moisture content and boundary condition in terms of confining and axial stresses of the laboratory M_r test resembled the in-situ roadbed condition where the FWD test was performed. Ji *et al.* (2014) observed a high scatter between their M_R and M_r values due to the variations in moisture content and boundary condition of the FWD and laboratory testing. Dawson *et al.* (2009) also acknowledged that the shift factor (a ratio of M_R to M_r) depended on the back-calculation software applied in the study but independent of the pavement type. A study by Nazzari and Mohammad (2010) for the Louisiana Department of Transportation and Development also concluded that the M_R values were significantly affected by the back-calculation method, and shift factors should be developed accordingly for respective methods. Mateos and Soares (2014) found that the back-calculation approach resulted in higher and more realistic modulus values of granular subgrade soils than that estimated using a calibrated nonlinear constitutive model from a repeated triaxial load testing.

The resilient modulus is back-calculated by matching FWD-measured pavement deflections with estimated deflections for each sensor location. The pavement deflection (D_e) for each sensor with respect to the plate load location can be estimated using Equation (1):

$$D_e = 1.5 \times p \times a \left\{ \frac{1}{M_R \sqrt{1 + \left(\frac{D}{a^3 \sqrt{E_p/M_R}} \right)^2}} + \frac{1 - \frac{1}{\sqrt{1 + \left(\frac{D}{a} \right)^2}}}{E_p} \right\} \quad (1)$$

where p is the pressure applied from the load plate to the pavement, a is the load plate radius, D is the total pavement thickness above subgrade, E_p is the elastic modulus of the pavement above subgrade and M_R is the back-calculated subgrade resilient modulus. For the entire deflection basin, a series of D_e values for all sensors will be estimated and compared with the respective measured deflections (D_m) obtained from the FWD test. Engineering judgement must be used during the back-calculation in interpreting the results (Irwin 2002). During the back-calculation process, the moduli of pavement layers are continuously adjusted until the theoretical or estimated deflection basin matches the measured deflection basin within a given tolerable root mean square error (RMSE) expressed in a percentage given by Equation (2).

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (D_{e_i} - D_{m_i})^2}{n}} \times 100\% \quad (2)$$

where n is the total number of estimated or measured pavement deflections. The M_R value will be determined during the iteration process based on the best match of deflections with the smallest RMSE. Generally, an acceptable range of the RMSE is between one and two per cent (WSDOT 2005). Based on this recommendation, one will strive to achieve the lowest possible RMSE during each back-calculation. However, the back-calculation having the lowest RMSE may not necessarily generate realistic resilient moduli. In fact, Seeds *et al.* (2000) recommended that any suggested thresholds for RMSE should be used cautiously, and engineering judgement should be used to determine if the back-calculated resilient moduli are reasonable. Mehta and Roque (2003) also acknowledged that a good fit between the measured and estimated deflections presented in terms of a relatively low RMSE may not necessarily yield a reasonable modulus value. However, the assessment of back-calculated M_R values and other in-situ tests (e.g. Dynamic Cone Penetration [DCP]) was found to be effective in achieving reliable modulus values (Oh *et al.* 2012). This limitation entails the challenge with the implementation of the FWD approach for facilitating the MEPDG Level 1 design that yields the most accurate pavement performance prediction.

Recognising the limitations associated with the current back-calculation procedure, a systematic protocol for back-calculating subgrade resilient modulus was developed as part of the comprehensive test program in Wyoming. The protocol was developed based on the most widely used back-calculation software program, MODCOMP6 with the MODTAG interface. Also, the back-calculation protocol was developed for a flexible pavement with a plane section, a granular crushed base and regular deflection basins that cover most pavement sections in the state

of Wyoming. Although the protocol was developed specifically for application in Wyoming, it can be similarly adopted by other national and international transportation agencies.

2. A comprehensive test program

Using a final embankment soil classification provided by the Wyoming Department of Transportation (WYDOT), 12 locations of existing pavements, denoted as Test Locations 1 through 12 shown in Figure 1, were identified for a comprehensive test program. Each location has three test sites, resulting in 36 test sites. These sites were selected to cover all representative soil types for subgrade materials in the state of Wyoming. Table 1 summarises the test location number corresponding to the order in which these sites were tested. The location refers to the closest city. A project number was assigned to each test number based on the WYDOT identification system. Three test sites per location were labelled as A, B and C.

In consideration of the safety of site crews when conducting the field testing on two-lane roadways, temperature holes were drilled near the shoulder at a mile marker representing the mid-point of the test site. Based on the pavement thickness identified during soil sampling, the number of temperature holes

and hole depths were determined in accordance with the LTPP manual for FWD measurements (Schmalzer 2006). To measure only pavement temperature, the last temperature hole was drilled to roughly 12.5 mm (0.5 in.) above the bottom of the pavement layer. These holes were filled with mineral oils to provide thermal conductivity between the pavement and a thermometer. The holes were covered with duct tapes, and thermometers were inserted into all holes to measure the temperature fluctuation at different depths as illustrated in Figure 2. The temperature at each depth was recorded every 10 minutes till the completion of the FWD test. The average mid-depth temperature of the asphalt pavement layer at each test site is summarised in Table 1.

FWD tests were conducted at each test site in accordance with the LTPP procedure (Schmalzer 2006). The FWD test was performed using a KUAB FWD with an eight-sensor set-up to record deflection measurements for four target loads of 26.7, 40, 53.4 and 71.2 kN (6, 9, 12 and 16 kips). These four load levels corresponded to four drop heights numbered 1 to 4. At each test station, the drop sequence adopted from the LTPP procedure is as follows:

- (1) three seating drops at drop height 3 corresponding to the target load of 53.4 kN,

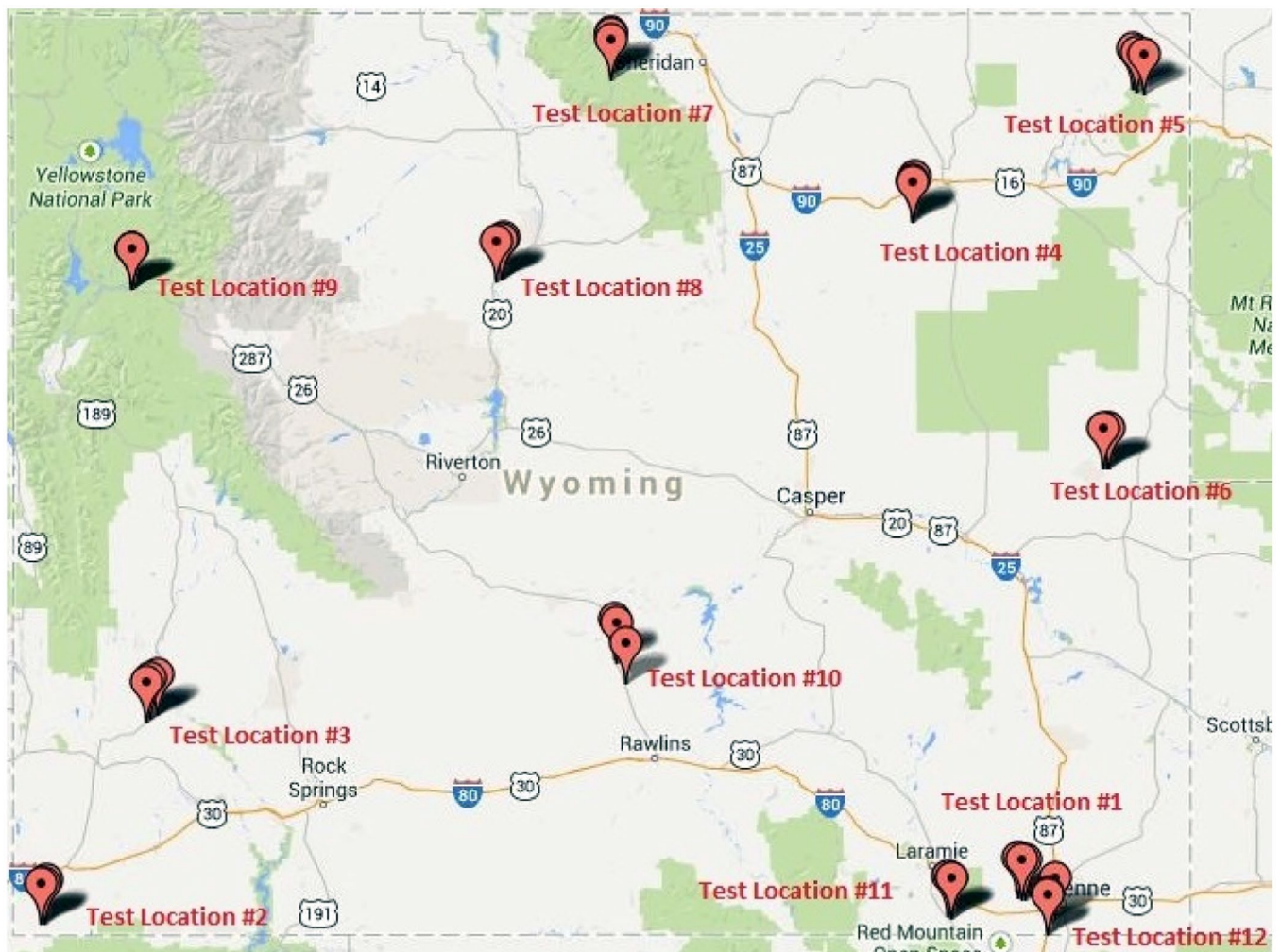


Figure 1. Twelve test locations or 36 test sites in the state of Wyoming. (Source: Author).

Table 1. Summary of 12 locations and 36 test sites.

Test loc.	Project name	Proj. no.	Test dates	Site	Asphalt/base thk. (cm)	Mid-depth temp (C)	Subgrade				
							Soil type	R	ω_{opt} (%)	γ_{d-max} (kN/m ³)	PI
1	Happy Jack Road (WYO 210)	0107	5/28/13 to 5/30/13	A ⁽⁴⁾	30.4/24.1	21.0	A-6	14	11.2	19.03	15
				B	30.4/24.1	23.3	A-4	47	23.2	14.69	15
				C	30.4/24.1	26.4	A-2-4	19	21.1	15.76	8
2	Evanston South (WYO 150)	2100	6/4/13	A ⁽³⁾	N/A	N/A	A-1-B	73	6.1	20.88	N/A
				B ⁽³⁾	N/A	N/A	N/A	N/A	N/A	N/A	N/A
				C ⁽³⁾	N/A	N/A	A-1-B	55	7.5	20.35	N/A
3	Kemmerer – La Barge (WYO 189)	0P11	6/5/13	A	33/24.1	25.3	A-6	10	14.7	17.80	19
				B	16.5/17.8	23.1	A-7-6	12	17	16.48	28
				C	15.2/30.4	29.6	A-7-6	15	17	16.64	22
4	Gillette – Pine Tree (WYO 50)	0300	6/11/13	A	10.2/30.4	22.9	A-6	18	16.4	17.19	21
				B	16/5.30.4	25.9	A-4	43	12.8	18.06	8
				C	12.7/33	30.8	A-6	10	15.3	17.62	17
5	Aladdin – Hulett (WYO 24)	0601	6/12/13	A	15.2/40.6	31.5	A-2-4	67	8.3	18.42	N/A
				B	15.2/45.7	27.9	A-2-4	61	6.6	15.82	N/A
				C	15.2/30.4	26.3	A-6	18	15.6	17.08	17
6	Lance Creek (WYO 270)	1401	6/13/13	A	10.2/25.4	24.7	A-7-6	13	18.5	15.62	40
				B	12.7/33	25.8	A-7-6	11	23.4	14.74	39
				C	12.7/27.9	26.8	A-7-6	13	28.4	14.21	26
7	Burgess Junction (US 14)	0N37	6/18/13	A	15.2/30.4	20.6	A-1-B	76	8.2	19.88	N/A
				B	15.2/30.4	19.1	A-1-B	72	6.1	20.04	N/A
				C	12.7/22.9	20.0	A-1-B	75	6.3	20.35	1
8	Thermopolis – Worland (US 20)	0N34	6/19/13	A ⁽⁵⁾	27.9/33	30.1	A-2-4	74	12.2	18.35	N/A
				B ⁽⁵⁾	25.4/30.4	27.2	A-4	47	10.9	18.87	3
				C	22.9/25.4	25.6	A-4	26	11.7	18.86	2
9	Moran Junction (US 26)	0N30	6/25/13	A	10.2/15.2	28.1	A-6	14	14.7	17.88	8
				B	10.2/15.2	26.7	A-1-A	65	6.4	20.29	1
				C	10.2/15.2	31.3	A-4	35	11.8	18.81	2
10	Lamont – Muddy Gap (WYO 789)	0N21	6/26/13	A ⁽²⁾	20.3/22.9	30.2	A-1-B	73	7.8	18.94	N/A
				B ⁽²⁾	20.3/17.8	32.3	A-6	12	14.9	17.47	14
				C	17.8/30.4	26.6	A-6	12	13.5	18.35	19
11	Laramie – CO. St. Line (US 287)	0N23	7/11/13	A	12.7/25.4	24.8	A-1-B	79	6.3	19.74	N/A
				B	12.7/25.4	31.9	A-1-B	75	5.2	19.89	N/A
				C	12.7/25.4	31.7	A-2-4	59	8.5	19.34	4
12	Cheyenne – CO. St. Line (I-25)	I025	7/12/13	A ⁽¹⁾	N/A	N/A	A-1-B	86	6.6	20.30	N/A
				B ⁽¹⁾	N/A	N/A	A-6	22	21.1	16.66	18
				C ⁽¹⁾	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Note. Loc. – Location; Proj. – Project; N/A – Not available; Thk. – Thickness; Temp – Temperature; R – R value; ω_{opt} – Optimum moisture content based on standard Proctor test; γ_{d-max} – Maximum dry unit weight based on standard Proctor test; PI – Plasticity index; CO. St. – Colorado State.

¹Excluded from back-calculation because of rigid pavement;

²Excluded from back-calculation because of cement-treated base;

³Excluded from back-calculation because of unusual stiff granular subgrade encountered during field test;

⁴Excluded from back-calculation because of super-elevation;

⁵Excluded from back-calculation because of anomalous deflection basins.

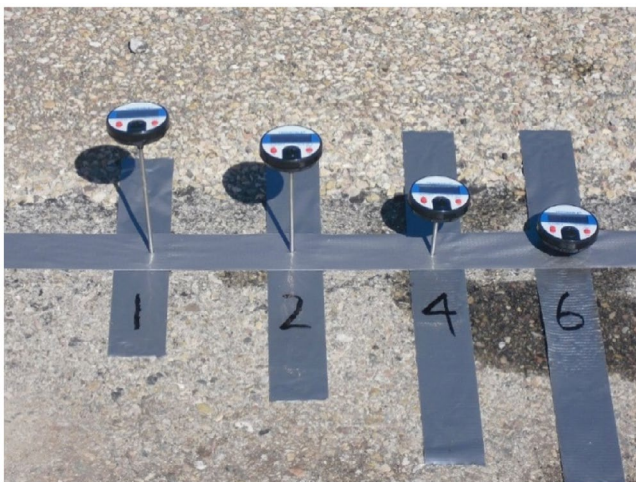


Figure 2. Temperature gradient set-up at 25 mm (1 in.), 50 mm (2 in.), 100 mm (4 in.) and 150 mm (6 in.) depths.

- (2) four drops at drop height 1 corresponding to the target load of 26.7 kN with a full-time history recorded on the last drop,
- (3) four drops at drop height 2 corresponding to the target load of 40 kN with a full-time history recorded on the last drop,
- (4) four drops at height 3 corresponding to the target load of 53.4 kN with a full-time history recorded on the last drop, and
- (5) four drops at drop height 4 corresponding to the target load of 71.2 kN with a full-time history recorded on the last drop.

FWD tests were performed at 15 stations over a 213.4 m (700 ft) test section per site. The interval between each test station was 15.2 m (50 feet). The distance was accurately measured using a distance measuring instrument (DMI). The FWD test began at Station 0, 106.7 m (350 ft) before the mile marker. After completing the test sequence, the FWD test was repeated at the next

test station and eventually completed at the last station, 106.7 m (350 ft) after the mile marker, as illustrated in Figure 3. Detailed FWD testing information can be found in Hellrung (2015).

A destructive testing was conducted at each test site to determine pavement thickness and collect subgrade samples. At the centre of each test site, 150-mm (6-in.) bore holes were drilled through the asphalt and base layers to determine layer thicknesses and material types. Upon reaching the subgrade, DCP tests and Standard Penetration Tests were conducted. Undisturbed subgrade samples were collected in thin-walled Shelby tubes for the measurement of in-situ subgrade properties. Disturbed subgrade soils were collected in bags for soil classification and other soil laboratory tests, notably the R-value test and laboratory resilient modulus test performed in accordance with the modified AASHTO T-307 (Henrichs 2015). Table 1 summarises the thicknesses of the asphalt and base layers, subgrade soil types and some subgrade soil properties. Detailed laboratory testing and results of distress surveys conducted at each test site can be found in theses by Henrichs (2015) and Hutson (2015). Furthermore, an electronic WYoming MEPDG Database was developed by Hutson *et al.* (2015) to efficiently compile all test results for the local calibration of MEPDG.

3. Back-calculation protocol

3.1. Pre-analysis

Criteria for test site selection, asphalt temperature correction and material seed modulus selection were proposed prior to conducting a back-calculation analysis. Since flexible pavement and granular crushed base are the most widely used pavement materials in the state, only sites with this combination were considered for the back-calculation. Hence, test location No. 12 in Cheyenne with rigid pavements and sites A and B of test location No. 10 with cement-treated bases were excluded. To ensure all test sites had similar pavement elevation characteristics, test sites on flat plane sections were only considered. This decision was attributed to the difficulty in collecting reliable deflection data on a super-elevated roadway section experienced during FWD testing in this project. Thus, site A of test location No. 1 was excluded.

To analyse all test sites under a uniform temperature condition, temperature correction was applied directly to FWD

deflection data instead of back-calculated resilient moduli. Particularly, asphalt temperature adjustment factor (ATAF) was applied to deflection data for sensor 0 which was directly beneath the FWD load plate. However, it is important to note that deflection data correction has the potential for errors with the deflection basin. The deflection basin should display a decreasing trend in deflection as sensors move further away from the loading plate. In some cases, after applying the ATAF, other sensors could display higher deflection readings than sensor 0. Thus, it is important to evaluate the deflection basins after applying the ATAF. Test sites with anomalous deflection basins, sites A and B of test location No. 8 were eliminated.

One of the challenges associated with the back-calculation process was the selection of an appropriate seed modulus for each layer of a pavement structure. Fwa and Rani (2005) acknowledged that the seed modulus could have significant impacts on the performance of back-calculation software and the final solutions of the M_R values. Since all FWD tests were performed during the summer when the asphalt layers were hot, a seed modulus of 2413 MPa (350,000 psi) was recommended. A 296-MPa (43,000-psi) seed modulus of granular crushed base layers that corresponds to the minimum R-value of 75 required by WYDOT was recommended (AASHTO 1993). The seed moduli of the subgrade soils, as summarised in Table 2, were selected as the typical values recommended in the NCHRP Report 1-37A (2004). However, if measured moduli are available, they are recommended to be used as the seed moduli in the back-calculation of resilient moduli.

3.2. Back-calculation approaches

After completing the pre-analysis, two back-calculation approaches were implemented to determine realistic back-calculated subgrade resilient moduli.

3.2.1. Seed value adjustment

One of the simplest approaches in the back-calculation process is consecutively adjusting the seed modulus from the upper to the lower layers to improve the matching between calculated and measured deflection basins. Instead of selecting an arbitrary seed modulus for each pavement layer, initiating the back-calculation process with seed moduli that represent local material

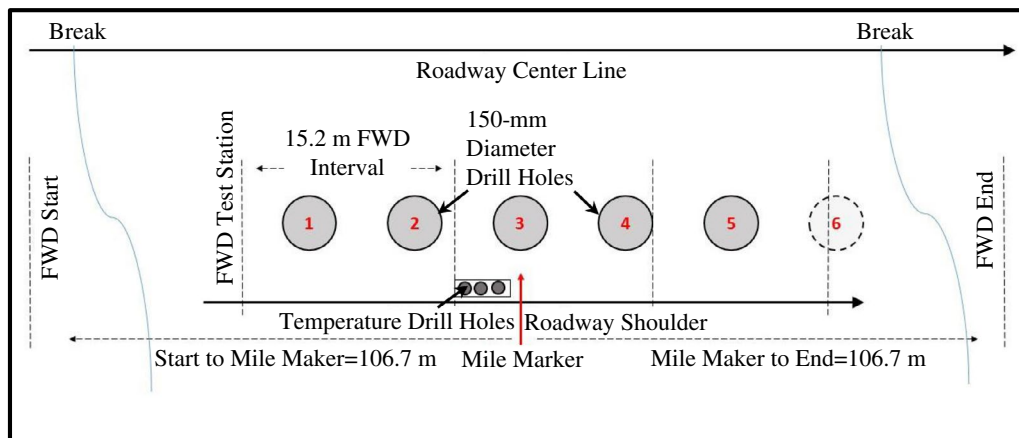


Figure 3. Illustration of a typical test site layout

Table 2. Summary of results from back-calculation analysis.

Test loc.	Proj. no.	Site	Asphalt M_R (MPa)	Base M_R (MPa)	Soil Type	Seed M_R^* (MPa)	Subgrade		RMSE (%)	Level
							Upper Subgrade M_R (MPa)	Lower Subgrade M_R (MPa)		
1	0107	B	5444.9	310.3	A-4	165.5	240.9	151.9	4.1	C
		C	1718.4	103.4	A-2-4	220.6	109.6	79.7	10.4	C
3	0P11	A	2789.5	117.2	A-6	117.2	128.2	115.7	8.2	B
		B	2831.7	151.7	A-7-6	55.2	139.4	88.6	3.2	A
		C	2164.3	82.7	A-7-6	55.2	106.1	61.2	7.7	B
4	0300	A	4816.8	82.7	A-6	117.2	77.6	66.3	8.3	B
		B	3517.9	117.2	A-4	165.5	114.9	81.6	5.5	A
		C	1870.8	82.7	A-6	117.2	83.8	70.2	8.0	B
5	0601	A	5349.3	448.2	A-2-4	220.6	375.7	202.4	5.1	C
		B	7457.9	482.6	A-2-4	220.6	405.7	199.4	4.6	C
		C	2128.2	151.7	A-6	117.2	230.0	131.1	3.5	A
6	1401	A	3986.3	82.7	A-7-6	55.2	71.7	51.9	6.3	A
		B	2906.5	82.7	A-7-6	55.2	99.9	53.4	4.6	A
		C	2822.7	82.7	A-7-6	55.2	136.5	94.8	8.0	B
7	0N37	A	2495.7	151.7	A-1-B	262.0	218.7	171.3	3.6	A
		B	2672.3	206.8	A-1-B	262.0	162.4	133.3	3.0	A
		C	4514.6	82.7	A-1-B	262.0	252.2	127.3	5.3	A
8	0N34	C	7215.6	186.2	A-4	165.5	109.0	113.9	4.3	C
9	0N30	A	2789.6	117.2	A-6	117.2	128.2	115.7	8.2	B
		B	5263.7	103.4	A-1-A	275.8	261.3	170.8	2.7	C
		C	2808.6	82.7	A-4	165.5	93.6	85.7	9.1	B
10	0N21	C	4602.7	241.3	A-6	117.2	194.9	99.1	2.5	A
11	0N23	A	5471.1	379.2	A-1-B	262.0	226.9	175.8	1.9	C
		B	4780.7	151.7	A-2-4	220.6	216.1	170.7	3.2	A
		C	5839.4	262.0	A-1-B	262.0	313.9	195.3	3.9	C

Note. Loc. – Location; Proj. – Project; M_R – Back-calculated resilient modulus; RMSE – Root mean square error.

*Recommended in the NCHRP Report 1-37A (2004).

characteristics will minimise the computational duration to achieve a best match between measured and estimated deflection basins. Nevertheless, changing the seed value has little effect on the final back-calculation results.

3.2.2. Pavement structural model adjustment

All pavement sections were modelled with an asphalt concrete surface layer followed by a granular crushed base and two subgrade layers. The subgrade was separated into two layers recognising that the upper subgrade had been compacted during construction as well as subjected to seasonal changes due to weather. The thicknesses of the asphalt and base layers were determined during the test program. Although a visible separation between subgrades could not be easily identified, the thickness of the upper subgrade was set to 610 mm (24 in.), a value recommended in the MODTAG Users Guide (Virginia Department of Transportation [VDOT] 2007). Although the two-layer model significantly reduces the average RMSE, extreme differences in resilient moduli between the base and subgrade materials were recognised with subgrades having higher resilient moduli.

In an effort to combat the issue of extreme differences in base and subgrade back-calculated moduli, a fixed-layer approach was utilised by fixing the base layer modulus, because the base layer at each site embodied similar characteristics of a typical granular crushed base. This approach shows dramatic improvements in achieving realistic layer moduli and reducing RMSE.

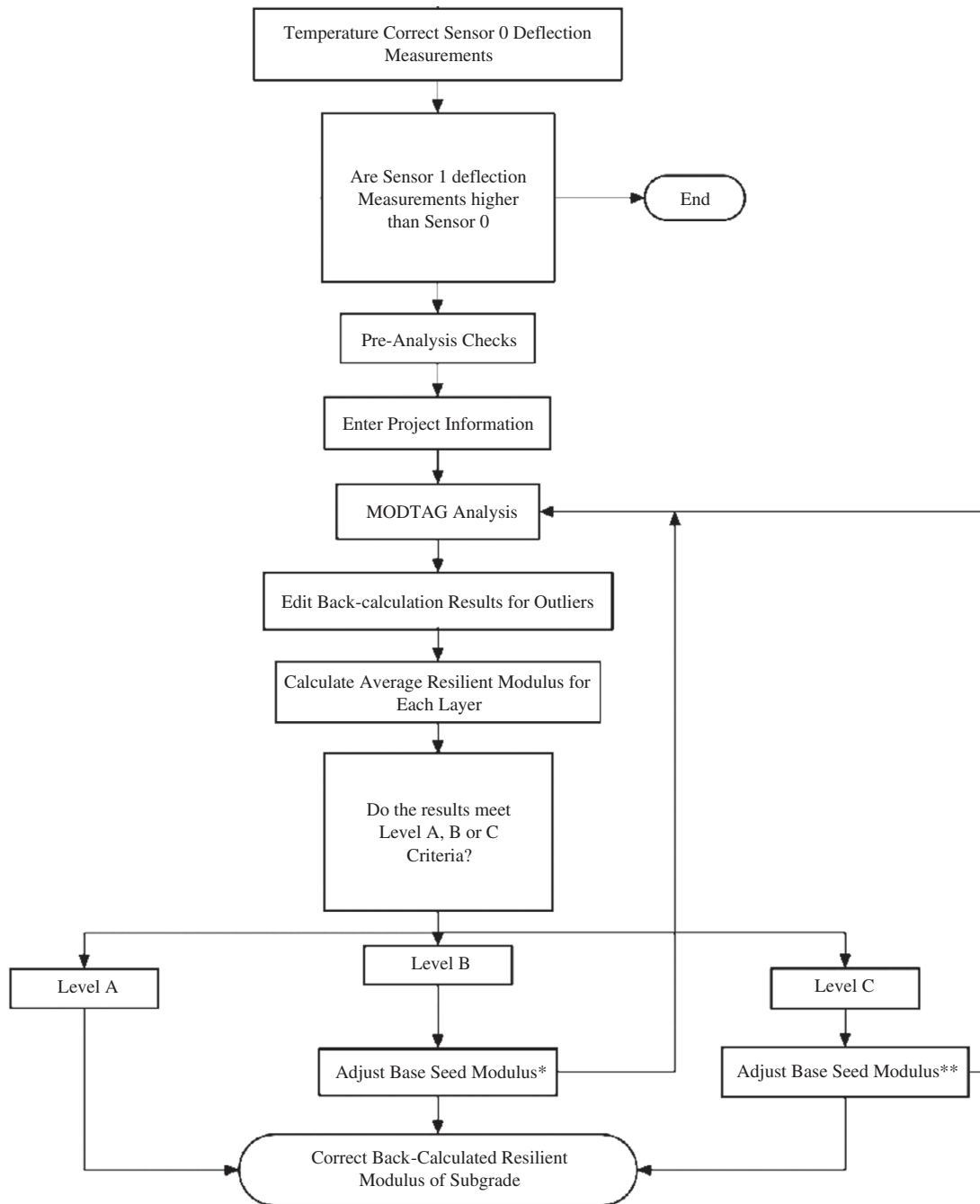
3.3. Back-calculation analysis

One of the challenges encountered during the analysis phase was the selection of four FWD drops for the back-calculation

analysis. Since the back-calculated modulus for each drop height was similar to that from other drops, an average modulus for each layer was determined. Another difficult task associated with the back-calculation is determining when to terminate the back-calculation process. To determine when to terminate the back-calculation process, three levels of analysis (A, B and C) based on specific criteria were established. These criteria were established based on literature review and typical material moduli ranges (VDOT 2007). Since the back-calculation analysis focuses on subgrade resilient modulus, a wider range of acceptable back-calculated M_R values for the asphalt layer, between 690 and 5171 MPa (100,000 and 750,000 psi), and for the base layer, between 69 and 552 MPa (10,000 and 80,000 psi), was allowed. Also, by expanding the acceptable range for the base material, more realistic asphalt and subgrade moduli were determined with smaller RMSEs. The lower and upper bounds of M_R values for subgrade soils were selected to be 27.6 MPa (4000 psi) based on the VDOT (2007) recommendation and 276 MPa (40,000 psi) from the NCHRP (2004), respectively. To satisfy Level A, which represents the most plausible M_R results, the asphalt modulus should range between 690 MPa (100,000 psi) and 5171 MPa (750,000 psi), the base modulus between 69 MPa (10,000 psi) and 552 MPa (80,000 psi), the subgrade modulus between 27.6 MPa (4000 psi) and 276 MPa (40,000 psi) and the RMSE should fall below 7%. If Level A is not satisfied, the back-calculation will be repeated based on Level B criteria. Level B results utilised the same bounds for the moduli, while the RMSE acceptance criterion was adjusted to accept results with RMSE below 11%. Likewise, if Level B is not satisfied, the back-calculation will be performed based on Level C criteria. Level C results eliminated the range of asphalt moduli while maintaining the same bounds for the base and subgrade as

Table 3. Summary of back-calculation analysis criteria.

Description	Level A	Level B	Level C
Back-calculated asphalt modulus (MPa)	690 to 5171	690 to 5171	N/A
Back-calculated base modulus (MPa)	69 to 552	69 to 552	69 to 552
Back-calculated subgrade modulus (MPa)	27.6 to 276	27.6 to 276	27.6 to 276
RMSE	Less than 7%	7% to 11%	Less than 11%

**Figure 4.** Back-calculation protocol flow chart.

*Continue to adjust the base seed modulus in order to achieve Level A criteria. If Level A criteria cannot be met, use best results meeting Level B criteria and then proceed to the next step. **Continue to adjust the base seed modulus in order to achieve Level A criteria. If Level A criteria cannot be met, adjust the base seed modulus in order to achieve Level B criteria. If Level A or B criteria cannot be met use best results meeting Level C criteria and then proceed to the next step.

required in Level A. The acceptable RMSE for Level C was below 11%. The back-calculation analysis criteria for these three levels are summarised in Table 3, and the proposed back-calculation

procedure is illustrated using a flow chart in Figure 4. If neither Levels A, B nor C criteria are met, the subgrade modulus will not be determined for MEPDG Level 1 design. Instead, MEPDG

Level 2 design using empirical relationships or MEPDG Level 3 design based on national defaulted values should be utilised to determine the resilient modulus.

4. Back-calculation results

Adopting the back-calculation protocol, the average back-calculated M_R values for all pavement layers and the 25 sites are summarised in Table 2. All test sites satisfied the Level A, B or C criteria. Ten test sites met Level A criteria, seven test sites met Level B criteria and eight test sites met Level C criteria. The average asphalt modulus of 3930 MPa (570,000 psi) is well within the typical range. The average base modulus of 174 MPa (25,198 psi) is closer to the lower bound of 69 MPa (10,000 psi) as the fixed-layer approach was utilised. The realistic difference in M_R values between upper and lower subgrade layers is evident. The average M_R value of 180 MPa (26,090 psi) for the upper subgrade is higher than 120 MPa (17,446 psi) for the lower subgrade.

5. Prediction models

5.1. Introduction

In order to estimate equivalent laboratory-measured subgrade resilient moduli from the back-calculated values, a plethora of prediction models were developed. During the laboratory M_r testing performed in accordance with the modified AASHTO T-307 (Henrichs, 2015), an average resilient modulus was measured for each of the 15 sequences at each test site. During the M_R back-calculation, resilient moduli were determined for the 610-mm (24-in.)-thick upper subgrade layer and the lower subgrade layer. To determine which combination of laboratory test sequence and subgrade layer yields the best prediction, a total of 60 linear prediction equations (i.e. 15 sequences \times 2 subgrade layers \times 2 model types) were developed to model the relationship between each sequence and subgrade layer. The performance of each prediction equation was evaluated based on the sum of squared errors (SSE).

The reason for using two model types stemmed from a combination of literature and current practice. Currently, most states use a zero intercept linear regression model to define the relationship between back-calculated M_R in the x -axis and laboratory-measured M_r values in the y -axis in terms of a single correlation factor (C) (i.e. $M_r = C \times M_R$). Hahn (1977) recommended that a zero intercept linear model should not be used when the values of an independent variable in the x -axis (i.e. back-calculated M_R) are relatively far from the origin at $x = 0$ unless the data clearly support the use of this model. Thus, a second, non-zero intercept linear regression model (i.e. $M_r = m \times M_R + b$) was also considered in this study, where m is a slope gradient and b is a non-zero y -intercept.

5.2. Data-sets

To investigate which model presents the best predictions, 30 data-sets were utilised in the study. Each data-set consisted of 25 laboratory-measured M_r values at each test sequence, and 25 back-calculated M_R values for a subgrade layer, for a total of 25 useable test sites (i.e. 1 data-set = 25 M_r laboratory test

results at a test sequence and back-calculated M_R at a subgrade layer). To cover all 15 M_r test sequences and 2 subgrade layers, 30 measured data-sets were considered in this study. Using the 25 data points from each data-set, both the zero intercept linear regression model and the non-zero intercept linear regression model were developed. The back-calculated M_R values of both subgrade layers for all 25 sites are summarised in Table 2, and the laboratory-measured M_r values from all 15 sequences for all 25 sites are summarised in Table 4. It is important to note that due to limited data-sets and the unfeasibility of performing soil sampling and classification during a non-destructive FWD test on site, regression models were developed for all subgrade types only.

5.3. Performance measures

Performance measures are required to compare the two model types. According to Hahn (1977), the coefficient of determination (R^2) value for the non-zero intercept linear model was calculated based on the proportion of variation, as measured by the sum of squares around the regression line. For the zero intercept linear model, R^2 is based on the proportion of variation around the origin. Since these two R^2 values are not comparable, R^2 was not used for comparison in this study.

To facilitate a true comparison, an SSE was utilised. The SSE is a measure of variation in the observations around a fitted regression line (Kutner *et al.* 2004). The SSE of each regression equation is determined by:

$$SSE = \sum_{i=1}^n (Y_{(i)} - \hat{Y}_{(i)})^2 \quad (3)$$

where Y_i is the laboratory-measured M_r and $\hat{Y}_{(i)}$ is the predicted M_r obtained from the regression equation. Since the typical resilient modulus of a subgrade material is in the tens to hundreds of megapascals and the differences are squared, the SSE can appear drastically increased. Thus, the SSE value of each regression or prediction equation should be used as a performance index by not judging its magnitude, but using it to identify which regression equations provide the best prediction.

5.4. Regression equation results

Statistical analysis software, R-Studio, was used to compare the laboratory-measured resilient moduli and the back-calculated values. For each data-set, two linear regression models (i.e. non-zero intercept linear model and zero intercept linear model) were determined by plotting the back-calculated M_R for the selected subgrade layer on the x -axis and the laboratory-measured M_r for the selected sequence on the y -axis. The linear model equations for the 30 data-sets on 15 sequences and 2 subgrade layers are summarised in Table 5. Both the y -intercept value (b) and slope gradient (m) are necessary for describing the non-zero intercept linear model. Only the slope gradient (C) or C -factor is needed for describing the zero intercept linear model. After the models were established, the SSE for each model was calculated using Equation (3) and plotted in Figure 5. The SSE values varied from 9347 (non-zero intercept linear model based on test sequence No. 14 and lower subgrade) to 81,104 (zero intercept

Table 4. Summary of laboratory-measured resilient moduli for all 15 test sequences and test site-specific sequences and resilient moduli.

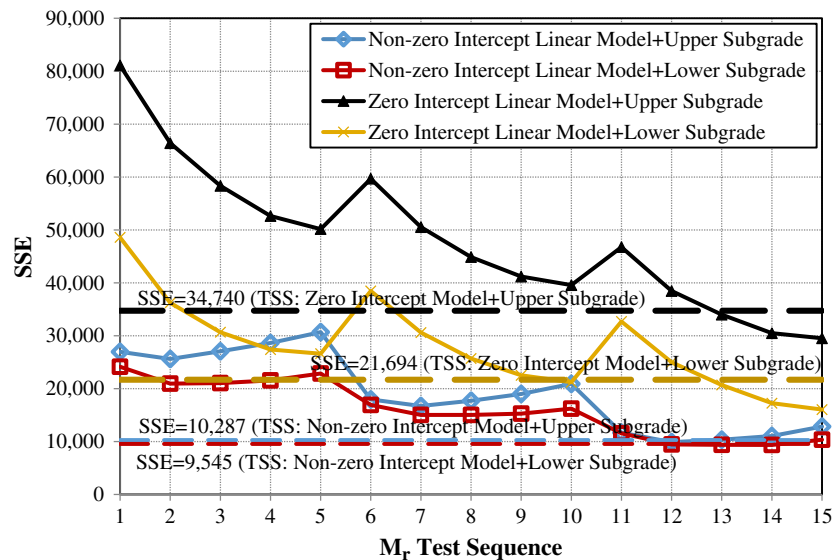
Project number	Laboratory-measured resilient modulus, M_r (MPa)															TSS	
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	SS	M_r (MPa)
0107-B	96.3	91.0	83.8	76.8	73.0	86.7	76.5	69.3	65.9	65.0	71.9	63.3	58.9	57.5	57.5	11	71.9
0107-C	93.0	88.9	80.4	73.5	68.3	80.3	76.0	69.8	65.8	62.9	69.1	64.7	60.5	57.1	55.4	11	69.1
0P11-A	139.6	135.5	123.1	116.4	110.9	123.8	117.6	113.3	107.1	105.2	105.1	101.7	100.1	95.6	94.6	11	105.1
0P11-B	105.0	98.5	90.7	84.9	80.9	94.9	88.6	83.0	79.2	76.9	84.8	79.2	75.7	73.3	70.9	13	75.7
0P11-C	106.0	103.9	95.4	89.4	83.8	95.2	92.0	87.2	84.2	80.6	84.9	82.1	79.6	76.1	73.3	12	82.1
0300-A	108.5	98.3	86.1	77.4	72.4	97.9	87.2	78.3	72.4	67.9	88.2	78.7	72.0	66.6	62.3	13	72.0
0300-B	161.9	154.2	149.5	140.1	138.2	136.3	128.8	124.3	123.0	124.6	112.1	102.2	101.1	100.7	103.6	12	102.2
0300-C	98.2	92.4	84.3	77.2	71.9	90.2	82.3	75.9	70.6	67.1	81.2	73.3	67.9	64.4	61.4	12	73.3
0601-A	89.4	121.9	121.5	121.6	121.5	101.3	97.2	96.9	99.9	105.0	81.4	74.7	75.1	80.3	85.3	12	81.7
0601-B	116.6	125.2	130.3	132.9	139.3	95.3	97.6	103.5	111.7	119.9	65.7	70.2	79.0	88.8	95.7	12	70.2
0601-C	141.3	130.5	122.5	114.8	110.6	153.5	132.4	122.3	114.3	107.1	142.6	118.4	109.6	103.0	97.0	12	118.4
1401-A	92.3	86.4	80.1	74.1	70.4	86.5	80.3	74.8	70.1	66.9	78.6	72.6	68.3	65.0	61.8	14	65.0
1401-B	61.9	54.5	48.5	43.7	40.4	59.9	51.5	45.2	41.7	38.4	52.6	45.8	41.3	37.6	35.5	13	41.3
1401-C	59.5	53.7	47.8	43.2	40.7	52.5	44.2	40.1	38.1	37.2	46.0	38.6	34.6	33.3	32.8	13	34.6
0N37-A	124.5	142.1	149.2	150.7	153.4	106.7	111.0	115.3	122.4	129.8	82.3	83.9	89.4	97.1	104.8	12	83.9
0N37-B	190.5	190.1	186.9	185.4	186.7	156.3	150.3	151.4	154.6	159.6	115.3	113.3	116.8	122.0	129.7	12	113.3
0N37-C	144.2	152.8	150.5	150.1	153.8	129.6	127.1	125.6	126.0	130.6	100.1	96.1	97.8	100.7	106.2	13	97.8
0N34-C	112.2	108.7	104.5	99.5	96.1	95.4	89.9	86.3	85.6	84.1	77.6	71.2	69.9	70.7	69.7	12	71.2
0N30-A	107.9	97.9	87.9	78.6	71.2	97.0	87.7	80.0	74.4	67.6	82.4	76.0	71.0	66.2	61.6	15	61.6
0N30-B	198.4	187.0	179.8	176.7	173.9	166.0	154.2	151.0	149.3	147.5	118.9	114.0	113.7	114.7	116.9	15	116.9
0N30-C	141.6	139.4	132.7	124.5	120.3	126.9	121.3	115.8	111.6	109.3	108.9	104.8	99.8	97.0	95.7	15	95.7
0N21-C	109.9	104.8	96.5	91.1	87.6	100.7	91.3	84.3	80.3	77.8	83.4	75.9	71.3	68.7	68.0	12	75.9
0N23-A	137.0	151.9	156.0	160.0	161.8	103.2	111.0	118.2	127.2	134.9	73.7	81.0	89.6	98.9	108.6	13	89.6
0N23-B	126.9	133.9	138.2	139.7	145.6	96.2	100.3	108.5	114.5	122.2	70.3	76.7	85.0	93.7	100.6	13	85.0
0N23-C	169.9	175.9	176.1	177.9	178.5	146.1	147.3	148.4	152.5	153.2	112.8	113.3	117.5	121.8	126.9	12	113.3

Note. S — General test sequence in accordance with the AASHTO T 307; SS — Test site-specific laboratory M_r test sequence; and TSS — Test site specific.

Table 5. Summary of linear regression equations for the 30 data-sets and two test site-specific data-sets.

Sequence	Subgrade	Non-zero intercept linear model		Zero intercept linear model
		b	m	C
1	Upper	103.6	0.099	0.558
1	Lower	85.9	0.295	0.914
2	Upper	89.9	0.172	0.570
2	Lower	67.9	0.44	0.929
3	Upper	78.7	0.208	0.557
3	Lower	53.8	0.518	0.906
4	Upper	69.0	0.239	0.545
4	Lower	41.9	0.582	0.885
5	Upper	62.1	0.266	0.542
5	Lower	33.6	0.635	0.878
6	Upper	90.9	0.090	0.494
6	Lower	80.6	0.221	0.802
7	Upper	81.8	0.111	0.474
7	Lower	68.5	0.276	0.770
8	Upper	73.4	0.141	0.467
8	Lower	56.7	0.349	0.758
9	Upper	66.3	0.175	0.469
9	Lower	46.7	0.424	0.761
10	Upper	60.8	0.205	0.475
10	Lower	38.8	0.489	0.769
11	Upper	83.4	0.023	0.393
11	Lower	79.8	0.064	0.640
12	Upper	75.3	0.042	0.376
12	Lower	68.4	0.120	0.614
13	Upper	68.4	0.074	0.378
13	Lower	58.4	0.195	0.616
14	Upper	62.1	0.111	0.386
14	Lower	48.8	0.276	0.628
15	Upper	57.5	0.142	0.397
15	Lower	41.5	0.346	0.645
TSS	Upper	67.6	0.078	0.378
TSS	Lower	56.4	0.210	0.616

Note. b – Y -intercept value for the non-zero intercept linear model; m – Slope gradient for the non-zero intercept linear model; and C – Slope gradient for zero intercept linear model or typically referred as C -factor.

**Figure 5.** Summary of sum of square errors (SSE).

linear model based on test sequence No. 1 and upper subgrade). The SSE values of the zero intercept linear models were generally higher than that of the non-zero intercept linear model. This observation implies that the non-zero intercept linear model provides a better prediction than the widely used zero intercept linear model. Lower SSE values were observed for both models

based on lower subgrade layers. The SSE values considering the lower subgrade decreased on average by 42 and 17% for the zero intercept linear model and the non-zero intercept linear model, respectively. Additionally, the SSE values for all models decreased with increasing M_r test sequences from 1 to 15. This decreasing trend suggests that the in-situ stress condition of a pavement

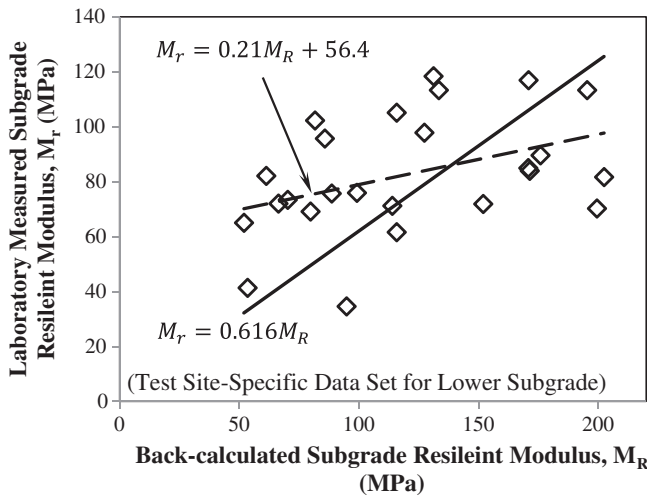


Figure 6. A relationship between back-calculated and laboratory-measured resilient modulus based on the test site-specific data-set for the lower subgrade layer.

structure can be best represented by stresses applied during higher M_r test sequences. This observation motivates the development of additional predictions based on a test site-specific M_r test sequence that mostly resembles the in-situ stress condition.

To improve the prediction, two additional data-sets were developed. Using the actual pavement layer thicknesses measured during the field test (see Table 1), the axial and confining stresses applied to the subgrade layer at each test site were determined (Henrichs 2015). The corresponding M_r test sequence (SS) with the axial and confining stresses closest to the actual stresses determined based on the pavement structure at each test site is summarised for each test site in Table 4. It is important to note that the test site-specific sequences vary from Nos. 11 to 15 which have a constant confining stress of 0.014 MPa (2 psi) and axial stresses from 0.014 to 0.069 MPa (2 to 10 psi), respectively. The M_r value corresponding to the test site-specific sequence for each site is also summarised in Table 4. The first test site-specific data-set consists of 25 test site-specific M_r and M_R values for the upper subgrade layer (see Table 2). Likewise, the second data-set consists of 25 test site-specific M_r and M_R values for the lower subgrade layer. Using the analysis procedure described previously, four regression or prediction equations developed for two subgrade layers utilising the two linear models are summarised in Table 5. The corresponding SSE values were also calculated and presented in Figure 5 for comparison. The non-zero intercept linear models for both subgrade layers with lower SSE values (9545 and 10,287) provided better predictions than the zero intercept linear models. Also, models based on the lower subgrade layer provide better predictions. Compared with the zero intercept linear models, a minimal difference is realised for the non-zero intercept linear models. Among all prediction equations for two linear regression models summarised in Table 5, the prediction equation that yields the lowest SSE value for each model is identified. For the non-zero intercept linear model, prediction Equation (4) based on the M_r test sequence No. 14 and lower subgrade layer yields the smallest SSE of 9347. Compared with the prediction Equation (4), prediction Equation (5) based on the test site-specific data-set for the lower subgrade yields a relatively higher SSE value of 9545.

Table 6. Summary of C-factors.

Agency	C-Factor
AASHTO	0.33
Colorado DOT	0.52
Idaho DOT	0.35
Missouri DOT	0.35
Montana DOT	0.50
Utah DOT	0.55 for fine-grained soil 0.67 for coarse-grained soil
Wyoming DOT	0.645

$$\hat{M}_r(\text{MPa}) = 0.276 \times M_R(\text{MPa}) + 48.8 \quad (4)$$

$$\hat{M}(\text{MPa}) = 0.210 \times M_R(\text{MPa}) + 56.4 \quad (5)$$

For the zero intercept linear model, prediction Equation (6) with the C-factor of 0.645 based on the M_r test sequence No. 15 and lower subgrade layer yields the lowest SSE of 16,036. If the test site-specific data-set for the lower subgrade is considered, prediction Equation (7) with the C-factor of 0.616 yields a relatively higher SSE of 21,694. Figure 6 shows the relationship between the back-calculated and laboratory-measured resilient modulus based on the test site-specific data-set for the lower subgrade layer.

$$\hat{M}_r(\text{MPa}) = 0.645 \times M_R(\text{MPa}) \quad (6)$$

$$\hat{M}_r(\text{MPa}) = 0.616 \times M_R(\text{MPa}) \quad (7)$$

5.5. Comparison of C-factors

Based on the AASHTO Road Test conducted in the late 1950s, AASHTO (1993) suggested using an adjustment factor (i.e. C-factor) of no more than 0.33. Having no local pavement data available during the initial development of the MEPDG, the C-factor of 0.33 was adopted as the default value. Rahim and George (2003) strongly suggested the need to re-evaluate this default value. To improve the pavement design efficiency and reflect local practices, several state DOTs initiated independent research to develop their respective locally calibrated MEPDGs. Outcomes of this effort led to the development of locally calibrated C-factors summarised in Table 6. Colorado DOT used the EVERCALC program for M_R back-calculation and found a C-factor of 0.52 for subgrade soils underneath a flexible pavement (ARA 2013). Idaho DOT conducted FWD testing at intervals of one test every tenth of a mile using two target loads of 40 and 53 kN (9 and 12 kips). Using MODULUS 6.0 software for the M_R back-calculation, the C-factor was determined to be 0.35 (ARA 2014). Likewise, the C-factor for flexible pavements determined for Missouri DOT was 0.35 (ARA 2009). Using FWD data collected at 11 sites throughout the state of Montana, M_R back-calculation was performed using MODULUS 6.0 software and the C-factor of 0.5 for subgrade soils under flexible pavement with a granular base was determined for Montana DOT (ARA 2007). According to a study by Utah DOT (2012), C-factors of 0.67 and 0.55 were determined for coarse-grained soils and fine-grained soils, respectively. The C-factor of 0.645 is recommended for the state of Wyoming which, although slightly higher, is comparable to the C-factors for the neighbouring states (i.e. 0.52 for Colorado, 0.55 and 0.67 for Utah and 0.50 for Montana).

This agreement could be attributed to similar geological and geotechnical conditions in the region. If the local calibration of MEPDG was not conducted and the AASHTO default C -factor of 0.33 was applied, the resilient modulus of subgrade soil will be underestimated, the pavement structure will be overdesigned, and the pavement cost will increase.

6. Conclusions

The following conclusions are drawn from this study:

- (1) A systematic back-calculation protocol based on the MODTAG Users Guide was developed to minimise the trial-and-error process in estimating the resilient modulus of subgrade soil. This protocol was specifically developed for roadways with flexible pavement and crushed base only. It was developed based on a series of pre-analysis checks, an interpolated mid-depth flexible pavement temperature, a pavement structure modelled using layer thicknesses and moduli, and three levels of acceptance criteria (A, B and C).
- (2) The results of the proposed back-calculation protocol show that the average asphalt modulus of 3930 MPa (570,000 psi) is well within the typical range. The average base modulus of 174 MPa (25,198 psi) is closer to the lower bound of 69 MPa (10,000 psi). The average M_R value of 180 MPa (26,090 psi) for the upper subgrade is higher than 120 MPa (17,446 psi) for the lower subgrade.
- (3) Two linear models: the non-zero intercept linear regression model and the zero intercept linear regression model were considered to correct back-calculated resilient modulus to laboratory-measured resilient modulus of a subgrade. The following conclusions were derived from the correlation study:
 - (a) The non-zero intercept linear model provides better correlations than the widely used zero intercept linear model.
 - (b) Better correlations were achieved when the back-calculated M_R values for lower subgrade layers were considered.
 - (c) Better correlations were obtained for data-sets based on higher laboratory M_r test sequences Nos. 11 to 15. This observation suggests that the in-situ stress condition of a pavement structure can be best represented by stresses applied at these higher M_r test sequences.
 - (d) For the non-zero intercept linear model, prediction Equation (4) with the slope gradient of 0.276 and y -intercept of 48.8 based on the M_r test sequence No. 14 and lower subgrade layer gives the best correlation.
 - (e) For the zero intercept linear model, prediction Equation (6) with the C -factor of 0.645 based on the M_r test sequence No. 15 and lower subgrade layer yields the best correlation. This C -factor of

0.645 recommended for Wyoming is comparable to the C -factors established by neighbouring states.

Although the results generated from this research were developed for WYDOT, the comprehensive field and laboratory test program, back-calculation protocol and methodology for correcting back-calculated resilient modulus value to laboratory measure resilient modulus can be adopted by other transportation agencies.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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